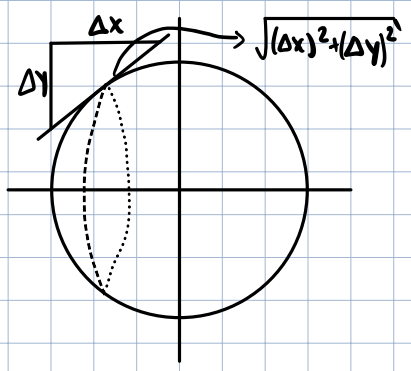
Surface Areas II

What should we have done?



$$\Delta s = \sqrt{(\Delta x)^2+(\Delta y)^2}$$

$$= \sqrt{(\Delta x)^2+\left(1+\left(\frac{\Delta y}{\Delta x}\right)^2\right)}$$

$$= \sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\cdot \Delta x$$

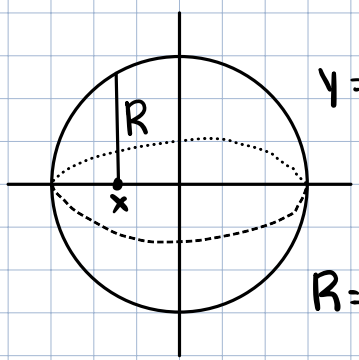
$\Delta x \to 0$

infinitesimal increment of arc length

$$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

area contributed by this bit of arc is $2\pi R\,ds$

$$SA = \int_a^b 2\pi R\,ds = \int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$



$$y = \sqrt{r^2-x^2}$$

$$R = \sqrt{r^2-x^2}, \quad -r \le x \le r$$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{r^2-x^2}}$$ (as in the arc-length calculation)

$$SA = \int_{-r}^{r} 2\pi \underbrace{\sqrt{r^2-x^2}}_{R} \underbrace{\sqrt{1+\left(\frac{-x}{\sqrt{r^2-x^2}}\right)^2}}_{ds}\,dx$$

$$= \int_{-r}^{r} 2\pi\sqrt{r^2-x^2}\sqrt{1+\frac{x^2}{r^2-x^2}}\,dx$$

$$= \int_{-r}^{r} 2\pi\sqrt{(r^2-x^2)\left(1+\frac{x^2}{r^2-x^2}\right)}\,dx$$

$$SA = \int_a^b 2\pi R\,ds$$

$$= 2\pi \int_{-r}^{r} \sqrt{r^2 - x^2 + \frac{x^2(r^2-x^2)}{r^2-x^2}}\, dx$$

$$= 2\pi \int_{-r}^{r} \sqrt{r^2 - x^2 + x^2}\, dx$$

$$= 2\pi \int_{-r}^{r} r\, dx$$

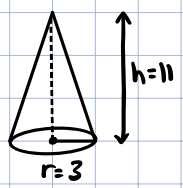
$$= 2\pi r x \Big|_{-r}^{r}$$

$$= 2\pi r \cdot r - 2\pi r(-r)$$

$$= 2\pi r^2 + 2\pi r^2$$

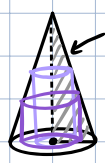
$$= 4\pi r^2$$

Right-circular cone with base radius 3 and height 11



Using Cylindrical shell method

$V = ?$



$y = 11 - \frac{11}{3}x$

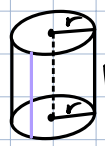
$0 \le x \le 3$

$0 \le y \le 11 - \frac{11}{3}x$

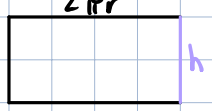
$h = 11 - \frac{11}{3}x$

$r = x - 0 = x$

SA of a cylinder of radius r & height h is



$h = 2\pi r h$



$$V = \int_0^3 2\pi r h\, dx$$

$$= \int_0^3 2\pi x(11 - \tfrac{11}{3}x)\, dx$$

$$= 2\pi \int_0^3 11x - \tfrac{11}{3}x^2\, dx$$

$$= 2\pi\left(\frac{11}{2}x^2 - \frac{11}{9}x^3\right)\Big|_0^3$$

$$= 2\pi\left(\frac{11\cdot 9}{2} - \frac{11\cdot 27}{9}\right) - 0$$

$$= 99\pi - 66\pi$$

$$= 33\pi$$

SA = ?

$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$$= \int_0^3 2\pi R\,ds$$

$$= \int_0^3 2\pi x\sqrt{1+\left(\frac{-11}{3}\right)^2}\,dx$$

$$= \int_0^3 2\pi x\sqrt{\frac{130}{9}}\,dx$$

$$= \frac{2\pi\sqrt{130}}{3}\int_0^3 x\,dx$$

$$= \frac{2\pi\sqrt{130}}{3}\left(\frac{x^2}{2}\right)\Big|_0^3$$

$$= \frac{2\pi\sqrt{130}}{3}\left(\frac{3^2}{2} - \frac{0^2}{2}\right)$$

$$= \frac{2\pi\sqrt{130}}{3}\left(\frac{9}{2}\right)$$

$$= 3\pi\sqrt{130}$$