Lecture 8

Feb 4^{th}, 2022

Integrating Rational Functions:

→ ratio of polynomials

ex/ $\int \frac{7x+4}{3x+2}dx = \int \frac{7x}{3x+2}dx + \int \frac{4}{3x+2}$ $\quad u=3x+2, du=3dx$

$\Rightarrow \int \frac{7\left(\frac{u-2}{3}\right)}{u}\cdot\frac{1}{3}du + \int \frac{4}{u}\cdot\frac{1}{3}du$

$= \frac{7}{3}\int \frac{u-2}{u}\cdot\frac{1}{3}du + \frac{4}{3}\int \frac{1}{u}du$

$= \frac{7}{9}\int(1-\frac{2}{u})du + \frac{4}{3}\int\frac{1}{u}du$

$= \frac{7}{9}(u-2\ln(u)) + \frac{4}{3}\ln(u) + C$

$= \frac{7}{9}u - \frac{14}{9}\ln(u) + \frac{4}{3}\ln(u) + C$

$= \frac{7}{9}u - \frac{2}{9}\ln(u) + C$

$= \frac{7}{9}(3x+2) - \frac{2}{9}\ln(3x+2) + C$

ex/ $\int \frac{3x+43}{x^2+8x+34}dx$

CTS: $x^2+8x+34$
$=(x+4)^2-16+34$
$=(x+4)^2+18$

$= \int \frac{3x+43}{(x+4)^2+18}$

$u=x+4, du=dx$
$\Rightarrow x=u-4$

$= \int \frac{3(u-4)+43}{u^2+18}du$

$= \int \frac{3u+31}{u^2+18}du = \int\frac{3u}{u^2+18}du + 31\int\frac{1}{u^2+18}du$

$w=u^2+18$
$dw=2udu$
$\Rightarrow udu=\frac{1}{2}dw$

$= \int \frac{3}{w}\cdot\frac{1}{2}dw + 31\int\frac{1}{u^2+18}du$

$= \frac{3}{2}\int\frac{1}{w}dw + 31\int\frac{1}{(\sqrt{18}\tan\theta)^2+18}\sqrt{18}\sec^2\theta d\theta$

$u=\sqrt{18}\tan\theta$
$du=\sqrt{18}\sec^2\theta\, d\theta$

$= \frac{3}{2}\ln(w) + 31\sqrt{18}\int\frac{\sec^2\theta}{18(\tan^2\theta+1)}d\theta = \frac{3}{2}\ln(w) + \frac{31\sqrt{18}}{18}\int 1\,d\theta$

$= \frac{3}{2}\ln(w) + \frac{31}{\sqrt{18}}\theta + C = \frac{3}{2}\ln((x+4)^2+18) + \frac{31}{\sqrt{18}}\arctan\left(\frac{x+4}{\sqrt{18}}\right) + C$

ex/ $\displaystyle\int \frac{x^3+x^2+2x+1}{x^2+4x+3}\,dx$

*rational functions are easier to integrate if degree of numerator is less than that of the denominator.

1) divide denominator into numerator

$$\begin{array}{r|l} & \quad x-3 \\ \hline x^2+4x+3 & x^3+x^2+2x+1 \\ & -(x^3+4x^2+3x) \downarrow \\ \hline & \quad -3x^2-x+1 \\ & -(-3x^2-12x-9) \\ \hline & \qquad 11x+10 \end{array}$$

$\Rightarrow x^3+x^2+2x+1$
$=(x^2+4x+3)(x-3)+(11x+10)$

$$=\int \frac{(x^2+4x+3)(x-3)+(11x+10)}{x^2+4x+3}\,dx$$

$$=\int (x-3)\,dx + \int \frac{11x+10}{x^2+4x+3}\,dx \quad \ldots$$