Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Winter 2021
Monstrous **Take-Home Final Examination** of Horror

Available on Blackboard from ~~12:00~~ 6:30 a.m. on Monday, 19 April.
Due on Blackboard by ~~11:59 p.m. on Wednesday, 21~~ 6:30 a.m. on Thursday, 22 April.

Submission: Scans or photos of handwritten work are entirely acceptable so long as they are legible and in some common format; solutions submitted as a single pdf are strongly preferred. If submission via Blackboard's Assignments module fails repeatedly, then (only as a *last* resort) email them to the instructor at: `sbilaniuk@trentu.ca`

Allowed aids: For this exam, you are permitted to use your textbook and all other course material, from this and any other mathematics course(s) you have taken or are taking now, but *you may not use any other sources or aids, nor give or receive any help*, except to ask the instructor to clarify questions and to use a calculator (any that you like).

Instructions: Do parts **X** and **Y**, and, if you wish, part **Z**. Please show all your work and justify all your answers. *If in doubt about something,* **ask!**

Part Xenomorph. Do *all four* (4) of **1**–**4**. *[Subtotal = 72]*

1. Compute $\dfrac{dy}{dx}$ as best you can in any *five* (5) of **a**–**f**. *[20 = 5 × 4 each]*

a. $y = \sqrt{\dfrac{x-1}{x+1}}$ **b.** $y = \displaystyle\int_0^{x^2} \sin(t)\, dt$ **c.** $y = \arctan\left(e^{2x}\right)$

d. $e^{xy} = x$ **e.** $y = \ln\left(x^2 - 1\right)$ **f.** $y = \dfrac{x^2-1}{x^4-1}$

2. Evaluate any *five* (5) of the integrals **a**–**f**. *[20 = 5 × 4 each]*

a. $\displaystyle\int \left(ue^{u+1} + 2^{u+2}\right) du$ **b.** $\displaystyle\int_0^2 \frac{v^2+39v-82}{v-2}\, dv$ **c.** $\displaystyle\int \frac{\tan(w)}{1-\sin^2(w)}\, dw$

d. $\displaystyle\int_0^{\pi/2} \frac{\sin(2x)}{\sqrt{1+\sin^2(x)}}\, dx$ **e.** $\displaystyle\int y \arctan\left(y^2\right) dy$ **f.** $\displaystyle\int_1^2 z^{40} \ln(z)\, dz$

3. Do any *five* (5) of **a**–**h**. *[20 = 5 × 4 each]*

a. Use the limit definition of the derivative to show that $\dfrac{d}{dx}x^4 = 4x^3$.

b. Compute $\displaystyle\lim_{x\to 0} x^{-1/2} \sin\left(x^2\right)$.

c. What is the minimum possible perimeter of a rectangle with area 25 m^2?

d. Sketch the solid obtained by revolving $y = 1 - x^2$, for $-1 \le x \le 1$, about the y-axis, and find the volume of this solid.

More exam on page 2!

Voila!

e. Find any and all vertical and horizontal asymptotes of $y = \arctan\left(x^2\right)$.

f. Find all the local maximum and minimum values of $y = xe^{-x^2}$ on $(-\infty, \infty)$.

g. Use the ε–δ definition of limits to verify that $\lim_{x \to 0} x \sin(x) = 0$.

h. Sketch the region whose bottom border is given by $y = \dfrac{x^2}{2}$ for $0 \leq x \leq 2$, and whose top border is given by $y = 2x$ for $0 \leq x \leq 1$ and by $y = 2$ for $1 \leq x \leq 2$, and find the area of this region.

4. Find the domain as well as any (and all) intercepts, vertical and horizontal asymptotes, intervals of increase, decrease and concavity, and maximum, minimum, and inflection points of $h(x) = \dfrac{x}{1 - x^2}$, and sketch its graph based on this information. *[12]*

Part Yeti. Do any *two* (2) of **5**–**7**. *[Subtotal = 28 = 2 × 14 each]*

5. A rectangle has its base on the x-axis and its top side runs from the line $y = x + 3$ on the left to the line $y = 3 - 3x$ on the right. Find the maximum area of such a rectangle.

6. Sketch the ellipse given by $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ and find its area.

Hint. The integral that would compute the area of an unit circle might be of interest.

7. Sand is poured onto a level floor at a constant rate of 100 L/min = 0.1 m^3/min, and at any given instant it forms a conical pile with the height equal to the radius of the base. Compute the rate of change of each of the following at the instant that the height of the cone is 1 m:

i. The height of the cone. *[5]*
ii. The area of the circular base of the cone. *[3]*
iii. The surface area of the rest of the cone. *[6]*

NOTE. The volume of a cone with base radius r and height h is $V = \dfrac{1}{3}\pi r^2 h$ and its surface area (not counting the base) is $S = \pi r \sqrt{r^2 + h^2}$.

[Total = 100]

Part Zombie. *Bonus problems!* If you feel like it, do one or both of these.

$\sqrt[2]{64}$. If $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots = \frac{\pi^2}{6}$, what does $\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} = 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots$ add up to? *[1]*

$\sqrt[3]{729}$. Write an original poem touching on calculus or mathematics in general. *[1]*

I HOPE THAT YOU ENJOYED THE COURSE. ENJOY THE SUMMER!