Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Winter 2020

Assignment #5
Speed of Convergence
Due on Thursday, 19 March.

Some parts of this assignment effectively require the use of mathematical software such as `Maple`, `Sagemath`, or `Mathematica`. Doing those parts entirely by hand, or even with the help of a calculator, will require considerable time and patience. Recall that Trent University has a (now 50-seat) site license for `Maple`, and note that significant chunks of the functionality of both `Sagemath` and `Mathematica` are available for free via web interfaces. (`Sagemath` is free, in any event.)

It's a fact that $\frac{1}{e} = e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = 1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \cdots$. (We'll see why this series adds up to $\frac{1}{e}$ once we do Taylor series.)

1. What is the least value of k such that $\sum_{n=0}^{k} \frac{(-1)^n}{n!} = 1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \cdots + \frac{(-1)^k}{k!}$ is within $0.0001 = 10^{-4}$ of $\frac{1}{e}$? Why? *[2]*

2. What is the least value of m such that $\sum_{n=0}^{k} \frac{(-1)^n}{n!} = 1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \cdots + \frac{(-1)^k}{k!}$ is within $0.0001 = 10^{-4}$ of $\frac{1}{e}$ for all $k \geq m$? Why? *[2]*

It's also a fact that $\frac{\pi}{4} = e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$. (Again, we'll see why this series adds up to $\frac{\pi}{4}$ once we do Taylor series.)

3. What is the least value of k such that $\sum_{n=0}^{k} \frac{(-1)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots + \frac{(-1)^k}{2k+1}$ is within $0.0001 = 10^{-4}$ of $\frac{\pi}{4}$? Why? *[2]*

The series $\sum_{n=0}^{\infty} \frac{2}{(4n+1)(4n+3)}$ also adds up to $\frac{\pi}{4}$. (To see why, answer question **4.**)

4. How are these two series adding up to $\frac{\pi}{4}$ related, besides having the same sum? *[2]*

5. What is the least value of k such that $\sum_{n=0}^{k} \frac{2}{(4n+1)(4n+3)}$ is within $0.0001 = 10^{-4}$ of $\frac{\pi}{4}$? Why? *[2]*