Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Winter 2020

Assignment #3
Plane Pursuit
Due on Thursday, 13 February.

A straight train track running north-south on a flat and otherwise featureless plain* meets a straight road running east-west at right angles. As the last car of a train heading north at 100 km/h passes this intersection, a drone† is flying west directly over the road at 200 km/h is 1 km east of the intersection. At this instant, the drone's controller decides to have it chase the train and thereafter keeps the drone headed directly towards the last car of the train until it catches up.

1. If the train and drone maintain their speeds of 100 km/h and 200 km/h, how far from the crossing does the drone catch up with the last car of the train? *[10]*

Hint: Find a differential equation describing the drone's path and take it from there. Suppose you set things up so that the x-axis runs along the road and the y-axis along the train tracks, with the origin at the intersection, all scaled in kilometres. If the path followed by the drone is the graph of $y = f(x)$, then $f(1) = 0$ with $\left.\dfrac{dy}{dx}\right|_{x=1} = f'(1) = 0$. The differential equation ought to be something like $2x\dfrac{d^2y}{dx^2} = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$ – it will be part of your job to explain this. When solving for $y = f(x)$, it will probably be a good idea to solve for $\dfrac{dy}{dx} = f'(x)$ first.

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* The plain is plainly a plain plane and a plane plain.

† Is the drone a plain plane flying over the plane plain?