Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Winter 2019
Assignment #6
Powerfully Serious Stuff
Due on Friday, 5 April.

1. Find a power series that is equal to $f(x) = \dfrac{1}{1 + x^2}$ when it converges and determine its radius and interval of convergence. *[3]*

   Hint: Think of $\dfrac{1}{1 + x^2}$ as the sum of a geometric series.

2. Use the power series you obtained in **1** to find a power series that is equal to $\arctan(x)$ when it converges and determine its radius and interval of convergence. *[3]*

   Hint: Integrate term-by-term.

3. Use the power series you obtained in **2** to find a series summing to π. How many terms of this series would you need to ensure that the partial sum is within 0.001 of π? *[4]*

   Hint: Hmm – what is arctan(1) equal to? For the second part, read up on the finer details of alternating series.

NOTE: The series you (hopefully!) obtained in **2** is often called *Gregory's series* after James Gregory, who rediscovered it in 1668. It had been previously discovered by Madhava of Sangamagrama (*c.* 1340 – *c.* 1425), a mathematician and astronomer from Kerala in southern India. He also obtained the series formula for π in **3**. Both the power series and the series formula for π were also rediscovered by Gottfried Leibniz in the 1670s.