Mathematics 1120H – Calculus I: Integrals and Series
TRENT UNIVERSITY, Summer 2018
Practice Final Examination

Time: *3 hours.* *Brought to you by* Стефан Біланюк.

Instructions: Do parts **A**, **B**, and **C**, and, if you wish, part **D**. Show all your work and justify all your answers. *If in doubt about something,* **ask!**

Aids: Any calculator; (all sides of) one aid sheet; one (1) brain (no neuron limit).

Part A. Do all four (4) of **1**–**4**.

1. Evaluate any *four* (4) of the integrals **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\int z\cos(2z)\,dz$ **b.** $\displaystyle\int_0^1 te^{-t^2}\,dt$ **c.** $\displaystyle\int \frac{x+1}{x^2+1}\,dx$

d. $\displaystyle\int_{-1}^1 \frac{1}{\sqrt{y^2+1}}\,dy$ **e.** $\displaystyle\int \frac{s^2}{s^2-1}\,ds$ **f.** $\displaystyle\int_0^{\pi/4} \frac{\sin^3(w)}{\cos^2(w)}\,dw$

2. Determine whether the series converges in any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{2^n}$ **b.** $\displaystyle\sum_{m=1}^{\infty} \frac{\sin(m\pi)}{\ln(m\pi)}$ **c.** $\displaystyle\sum_{\ell=2}^{\infty} e^{-\ell^2}$

d. $\displaystyle\sum_{k=3}^{\infty} \frac{k!\cdot 2^k}{3^k}$ **e.** $\displaystyle\sum_{j=4}^{\infty} \frac{j^2-j+1}{\sqrt{j^5+13}}$ **f.** $\displaystyle\sum_{i=5}^{\infty} \cos(i\pi)\sqrt{\left(\frac{1}{2}\right)^i}$

3. Do any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. Use the Right-Hand Rule or the Trapezoid Rule to approximate $\displaystyle\int_0^1 \left(1-x^2\right)\,dx$ to within $\frac{1}{2} = 0.5$ of the exact value.

b. Find the area of the finite region between $y = x^2$ and $y = x + 2$.

c. Suppose $a_1 = 1$ and $a_{n+1} = \dfrac{n+1}{n} a_n$. Compute $\displaystyle\lim_{n\to\infty} a_n$.

d. Find the volume of the solid obtained by revolving the region below $y = 2$ and above $y = 1$, for $1 \le x \le 2$, about the y-axis.

e. Suppose $\sigma(n) = \begin{cases} 1 & \text{if } n = 4k \text{ or } 4k+1 \text{ for some integer } k \\ -1 & \text{if } n = 4k+2 \text{ or } 4k+3 \text{ for some integer } k \end{cases}$. What function has $\displaystyle\sum_{n=0}^{\infty} \frac{\sigma(n)x^n}{n!}$ as its Taylor series at $a = 0$?

f. Find the Taylor series at $a = 0$ of $f(x) = e^{2x}$ and determine its interval of convergence.

4. Consider the region bounded by $y = 0$ and $y = \dfrac{1}{x}$ for $1 \le x < \infty$.

a. Find the area of this region. *[4]*

b. Find the volume of the solid obtained by revolving the region about the x-axis. *[8]*

Part B. Do either *one* (1) of **5** or **6**. *[14]*

5. Consider the piece of the parabola $y = \frac{1}{2}x^2$ for which $0 \leq x \leq 2$.

a. Find the arc-length of this piece. *[9]*

b. Find the area of the surface obtained by revolving this piece about the y-axis. *[5]*

6. The region below $y = -x^2 + 4x - 3$ and above $y = 0$ for $1 \leq x \leq 3$ is revolved about the line $x = -1$. Find the volume of the resulting solid. *[14]*

Part C. Do either *one* (1) of **7** or **8**. *[14]*

7. Find the Taylor series at $a = 0$ of $f(x) = \dfrac{2}{x+2}$ and determine its radius and interval of convergence.

8. Find the Taylor series at $a = 1$ of $f(x) = \dfrac{2}{1+x}$ and determine its radius and interval of convergence.

[Total = 100]

Part D. Bonus problems! If you feel like it and have the time, do one or both of these.

△. What does the infinite product $2 \prod_{n=1}^{\infty} \left[\frac{2n}{2n-1} \cdot \frac{2n}{2n+1} \right] = 2 \cdot \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdot \cdots$ amount to? *[1]*

□. Write a haiku (or several :-) touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

ENJOY THE REST OF YOUR SUMMER!