Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
Trent University, Summer 2023 (S61)
Solutions to Quiz #6
Meredith Meanders

Meredith decides to cycle from home at Point A to a scenic location at Point B and back by exactly the same route. The route in question has a mix of level ground and some gentle hills. On level ground Meredith cycles at 40 km/hr, going uphill at 30 km/hr, and going downhill at 60 km/hr. It takes Meredith 6 hours to make the return trip, not counting the time spent at Point B.

Please give your complete reasoning when answering the two questions below.

1. As best as you can determine based on the information provided, what is the distance between Point A and Point B along the route Meredith took? *[3]*

SOLUTION. I ~~stole~~ *took creative inspiration for* this problem from *Knot I* of Lewis Carroll's *A Tangled Tale*, which is a collection of mostly mathematical puzzles embedded in ten interlinked stories. What follows is his solution, modified to fit the setup above.

A level kilometer takes $\frac{1}{40}$ of an hour, uphill $\frac{1}{30}$, downhill $\frac{1}{60}$. Hence to go and return over the same kilometer, whether on the level or on the hill-side, takes $\frac{1}{20}$ of an hour. (That is, Meredith's average speed there and back over any part of the route is $2 \cdot 20 = 40$ km/hr.) Hence in 6 hours Meredith travels a total of $6 \cdot 40 = 240$ kilometers, 120 kilometers out and 120 kilometers back, *i.e.* the route Meredith took between Point A and Point B is 120 km long. □

2. As best as you can determine based on the information provided, how long after setting out from Point A did it take Meredith to reach Point B? *[2]*

SOLUTION. Here is Carroll's solution, modified to fit the setup given here.

If the 120 kilometers along the route had been all level, Meredith would have taken 3 hours; if all uphill, 4 hours; if all down hill, 2 hours. Thus it must have taken Meredith 3 ± 1 hours to reach Point B. □