Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Summer 2023 (S61)

Final Examination
19:00-22:00 in ENW 114 on Wednesday, 14 June.

Instructions: Do both of parts **I** and **II**, and, if you wish, part **III**. Please show all your work, justify all your answers, and simplify these where you reasonably can. When you are asked to do k of n questions, only the first k that are not crossed out will be marked. *If you have a question, or are in doubt about something,* **ask!**

Aids: Any calculator, as long as it can't communicate with other devices; (all sides of) one letter- or A4-size sheet; one natural intelligence.

Part I. Do all four (4) of **1**–**4**.

1. Compute $\frac{dy}{dx}$ as best you can in any four (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. $y = x\tan(x)$ **b.** $y = \frac{\cos(x)}{x}$ **c.** $y = \int_1^{x/2} \cos(t)\,dt$

d. $y = (x-3)^{10}$ **e.** $y = \ln(1+e^x)$ **f.** $y = \sin^2(\ln(x))$

2. Evaluate any four (4) of the integrals **a**–**f**. *[20 = 4 × 5 each]*

a. $\int \frac{x}{x^2+1}\,dx$ **b.** $\int_0^{e-1} \frac{x}{x+1}\,dx$ **c.** $\int_0^{\pi} x\cos(x)\,dx$

d. $\int \frac{x^2+x}{x+1}\,dx$ **e.** $\int \tan^2(x)\,dx$ **f.** $\int_0^1 2x^3 e^{x^2}\,dx$

3. Do any four (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. Compute $\lim_{x\to 0} \frac{x}{\tan(x)}$.

b. Use the ε–δ definition of limits to verify that $\lim_{x\to 2}(4x-7) = 1$.

c. At what point (x, y) does the graph of $y = e^x$ have a tangent line with slope 2?

d. Sketch the region between $y = x+2$ and $y = x^2$, for $-1 \le x \le 2$, and find its area.

e. Let $f(x) = \begin{cases} x\ln(x) & x > 0 \\ 0 & x \le 0 \end{cases}$. Determine whether $f(x)$ is continuous at $x = 0$.

f. Suppose $f'(x) = x^2$ and $f(1) = 1$. What is the function $f(x)$?

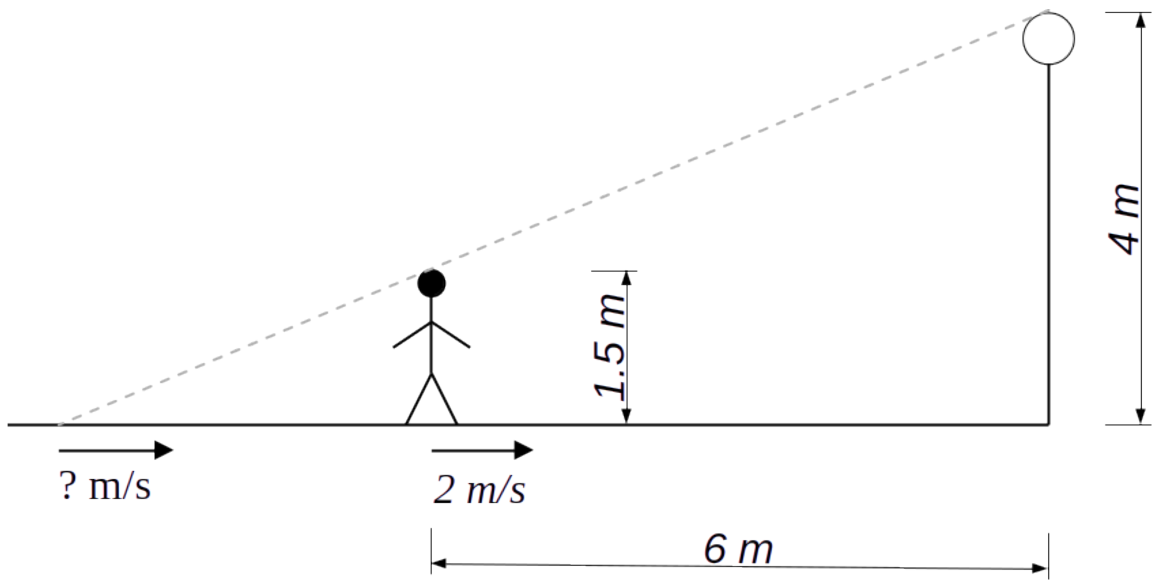
4. Find the domain, intercepts, vertical and horizontal asymptotes, intervals of increase and decrease, maximum and minimum points, intervals of concavity, and inflection points of $f(x) = \frac{x}{1+x^2}$. *[15]*

It's not over! Parts II and III are on page 2.

Part II. Do one (1) of **5–7**.

5. The region between $y = \sqrt{x}$ and $y = x^2$, for $0 \leq x \leq 1$, is revolved about the x-axis. Find the volume of the resulting solid. *[10]*

6. Stick Figure, who is 1.5 m tall, walks at 2 m/s on level ground at night, straight towards a 4 m tall lit up lamppost. How fast is the tip of Stick's shadow moving along the ground at the instant that Stick is 6 m from the lamppost? *[10]*

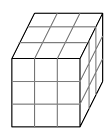


7. Find the maximum possible area of a rectangle whose corners are at $(x, 1 - x^2)$, $(-x, 1 - x^2)$, $(-x, 0)$, and $(x, 0)$, for some x with $0 \leq x \leq 1$. *[10]*

[Total = 85]

Part III. *Here be bonus points!* Do one or both of $\mathbf{2^3}$ and $\mathbf{3^2}$.

$\mathbf{2^3}$**.** A dangerously sharp tool is used to cut a cube with a side length of 3 *cm* into 27 smaller cubes with a side length of 1 *cm*. This can be done easily with six cuts. Can it be done with fewer? (Rearranging the pieces between cuts is allowed.) If so, explain how; if not, explain why not. *[1]*



$\mathbf{3^2}$**.** Write a haiku touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

Enjoy the rest of the summer!