

Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Summer 2021 (S62)

Assignment #5
Oddly Shaped

Due on Friday, 23 July.

Submission: Scanned or photographed solutions are fine, so long as they are legible. Please try to make sure that they are oriented correctly – if they are sideways or upside down, they're rather harder to mark online. Submission as a single pdf is strongly preferred, but other common formats are probably OK in a pinch. Please submit your solutions via Blackboard's Assignments module. If Blackboard does not acknowledge a successful upload, please try again. As a *last* resort, email your solutions to the instructor at: sbilaniuk@trentu.ca

Suppose we construct a two-dimensional shape as follows.

At stage 0 we have an equilateral triangle, consisting of three straight line segments of length 1 stuck together end-to-end.

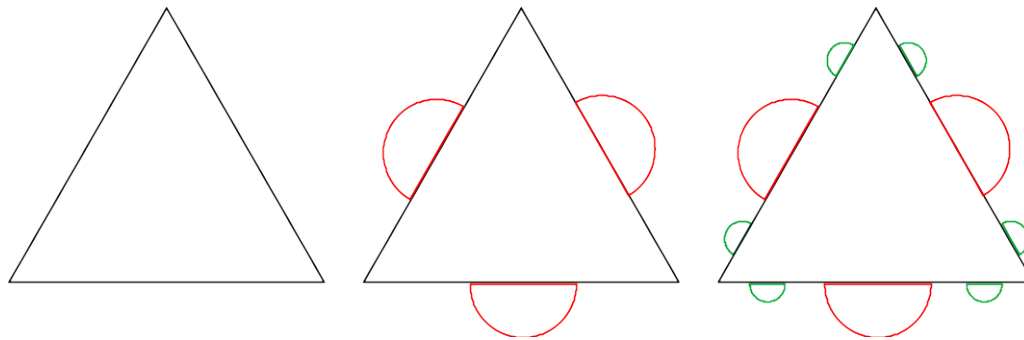
At stage 1 we attach a semi-circle on the outside of the triangle to the middle third of each side, with the base of each semi-circle being that middle third.

At stage 2 we attach a semi-circle on the outside of the triangle to the middle third of each straight piece of a side that is not already the base of a semi-circle, with the base of each new semi-circle being that middle third.

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In general, at stage $n > 0$ we attach a semi-circle on the outside of the triangle to the middle third of each straight piece of a side that is not already the base of a semi-circle, with the base of each new semi-circle being that middle third.

The shape we are interested in is the one we have after completing infinitely many stages, one for each $n \geq 0$. A crude sketch of stages 0 through 2 is given below.



Please answer both questions below, giving your reasoning in detail.

1. What is the length of the perimeter of the shape we have after infinitely many steps, not including the bases of any semi-circles? [5]
2. What is the total area of the shape we have after infinitely many steps? [5]

[Total = 10]