

Mathematics 1120H – Calculus II: Integrals and Series

TRENT UNIVERSITY, Summer 2021 (S62)

Assignment #4

Series Business

Due on Friday, 9 July.

Submission: Scanned or photographed solutions are fine, so long as they are legible. Please try to make sure that they are oriented correctly – if they are sideways or upside down, they're rather harder to mark online. Submission as a single pdf is strongly preferred, but other common formats are probably OK in a pinch. Please submit your solutions via Blackboard's Assignments module. If Blackboard does not acknowledge a successful upload, please try again. As a *last* resort, email your solutions to the instructor at: sbilaniuk@trentu.ca

Most of the time, finding the actual sum of a series is pretty hard, which is why we usually settle for determining whether the series converges, *i.e.* has a reasonable sum, or not. In this assignment, we will look at a few of the exceptions. Along with the usual tools of algebra, differentiation, and integration, we will rely on the one class of series that has a really nice summation formula, namely geometric series. The geometric series with first term a and common ratio r is the series $\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \dots$. As long as $|r| < 1$, or $a = 0$, this series is guaranteed to converge and adds up to $\frac{a}{1-r}$; if $|r| \geq 1$ and $a \neq 0$, it is guaranteed to diverge.

You may use the facts noted above, as well as those developed in the lectures and the textbook, in answering the questions below. You may also assume that you can safely differentiate and integrate series involving powers of x term-by-term when they converge.

1. What is the sum of the series $\sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \dots$ when it converges? For which values of x does it converge? [1]
2. What should the the sum of the series $\sum_{n=1}^{\infty} (-1)^n n x^{n-1} = -1 + 2x - 3x^2 + 4x^3 - \dots$ be when it converges? Why? [1.5]
3. What should the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ be when it converges? Why? [1.5]
4. What should the sum of $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \dots$ be, assuming it converges? [0.5]
5. What should the sum of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$ be, assuming it converges? [0.5]
6. Starting with the series $\sum_{n=0}^{\infty} (-1)^n x^{2n}$, give an (informal) chain of reasoning that computes the sum of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$, assuming it converges. [5]

[Total = 10]