Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Winter 2020

Assignment #3
Exponential and Differential
Due on Friday, 10 July.

Please submit your solutions using Blackboard's assignment module. If that fails, please email your solutions to the instructor (`sbilaniuk@trentu.ca`). Scans or photos of handwritten solutions are perfectly acceptable, so long as they are legible and in some common format. (Combined into a single pdf, for preference.)

Just in case you haven't seen it before, or have forgotten about it, the notation $n!$ is a shorthand for the product of the first n positive integers, that is:

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

Thus $1! = 1$, $2! = 2 \cdot 1 = 2$, $3! = 3 \cdot 2 \cdot 1 = 6$, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$, and so on. $n!$ grows very quickly, faster than any exponential function with a constant base. (Stirling's Formula tells us that when n is large, $n!$ is approximately $\sqrt{2n\pi} \cdot \dfrac{n^n}{e^n}$.)

This notation is extended to $n = 0$ by defining $0! = 1$. This is mainly done to make various general formulas and expressions involving $n!$ (including the sum in question **2** below) behave nicely when $n = 0$. One could also justify this by observing that $n!$ counts the number of ways one can arrange n distinct objects in a row, and that there is only one way of arranging no objects at all ...

1. Suppose $y = f(x)$ satisfies the equation $\dfrac{dy}{dx} = y$. Show that $f(x) = Ke^x$ for some constant K. *[5]*

2. Suppose $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$. Use **1** (and just a bit more) to show that $f(x) = e^x$. *[5]*

NOTE. For the sake of this assignment, you may assume that the sum $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$ makes sense no matter what the value of x is. We'll see exactly what this means and how to check it is so later in the course. For now, just think of the sum as a polynomial of infinite degree.