Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals
TRENT UNIVERSITY, Summer 2018
Slutions to Assignment #2
Plotting curves with `Maple`

Before attempting the questions below, please read through Chapter 1 of the text for the basics of graphing functions in Cartesian coordinates, and (at least) skim through Chapter 10 for the basics of parametric curves and polar coordinates, respectively. Some basics of graphing in Cartesian coordinates, using trigonometric functions, and so on, can also be found in the Academic Skills pamphlet *Formula for Success*. The basic definitions of how parametric curves and polar coordinates work are given in this assignment for your convenience, but you might want some additional explanations and examples. You should also read the handout *A very quick start with* `Maple` and play around with `Maple` a little. It might also be useful to skim though *Getting started with* `Maple 10` by Gilberto E. Urroz – read those parts concerned with plotting curves more closely! – and perhaps keep it handy as a reference. You can find links to these documents on the MATH 1110H web page at `euclid.trentu.ca/math/sb/1110H/` . `Maple`'s help facility may also come in handy, especially when trying to make out the intricacies of what the `plot` command and its options and variations do. Make use of each other and the instructor, too! Don't forget that while you may work together and look stuff up for the assignments. you should write and/or type up what you submit by yourself.

NOTE. *If you would rather use a comparable program other than* `Maple`*, you may do so.*

A curve is easy to graph, at least in principle, if it can be described by a function of x in Cartesian coordinates.
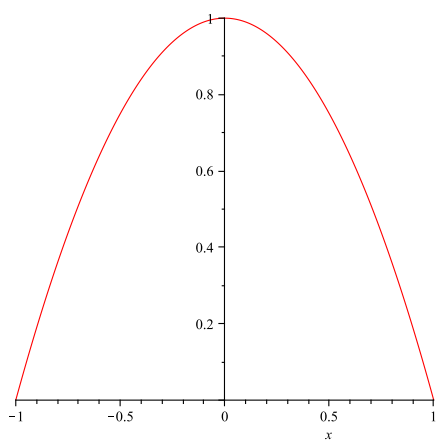
1. Use `Maple` to plot the curves defined by $y = x^2$, $y = 1 - x^2$, $y = \sqrt{x^2}$, and $y = \sqrt{1 - x^2}$, respectively, for $-1 \le x \le 1$ in each case. [Please submit a printout of your worksheet(s).] *[2]*

SOLUTION. Suitable uses of the `plot` command and their output are given below. [The graphs in this and later solutions have been shrunk to save some space, but are otherwise unaltered.]
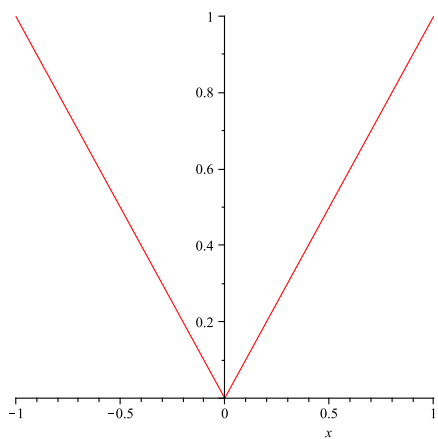
$>$ `plot(x^2, x = -1..1)`
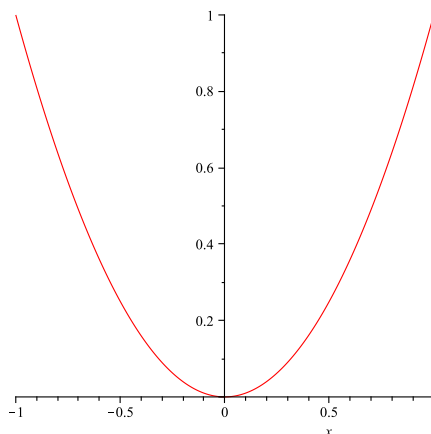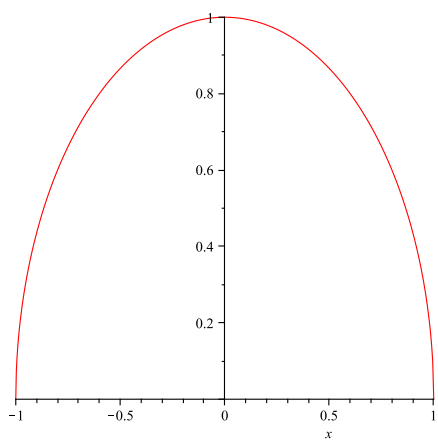
```
> plot(1-x^2, x = -1..1)
```



```
> plot(sqrt(x^2), x = -1..1)
```



```
> plot(sqrt(1-x^2), x = -1..1)
```

In many cases, a curve is difficult to break up into pieces that are defined by functions of x (or of y) and so is defined implicitly by an equation relating x and y; that is, the curve consists of all points (x, y) such that x and y satisfy the equation. One can, of course, also use implicit definitions to describe curves that can also be defined as the graphs of functions.

2. Use `Maple` to plot the curves implicitly defined by $x^2 + y^2 = 1$ for $y \geq 0$, $y^2 - x^2 = 0$ for $-1 \leq x \leq 1$, and $\left(x^2 + y^2\right)^2 + 4xy\left(x^2 + y^2\right) - 4y^2 = 0$ for all x and y satisfying the equation, respectively. [Please submit a printout of your worksheet(s).] *[2]*

SOLUTION. To use the `implicitplot` command, as well as `polarplot` in the solution to **4**, we first need to load the `plots` package:
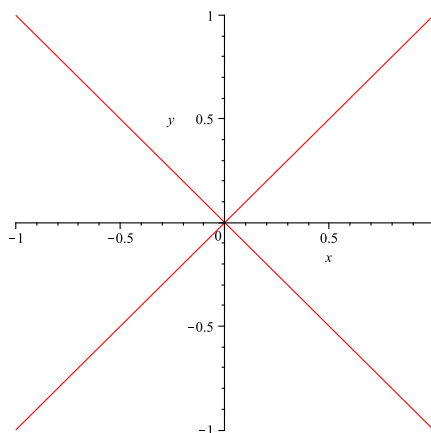
$>$ `with(plots)`

Now on with the show:

$>$ `implicitplot(x^2+y^2 = 1, x = -1..1, y = 0..1)`
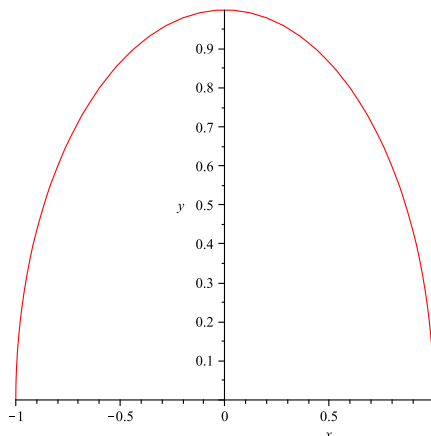
In the next plot we use the `gridrefine` parameter to improve the resolution of the graph. (Without that, the graph will be a little messed up near $(0,0)$.)

$>$ `implicitplot(x^2-y^2 = 0, x = -1..1, y = -1..1, gridrefine = 4)`

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For the next plot, the curve is unbounded, so we can't actually plot the whole thing. The bounds chosen for x and y are enough to show what the curve looks like:

```
> implicitplot((x^2-y^2)^2+4*x*y*(x^2-y^2)-4*y^2 = 0, x = -5..5,
  y = -5..5, gridrefine = 4)
```
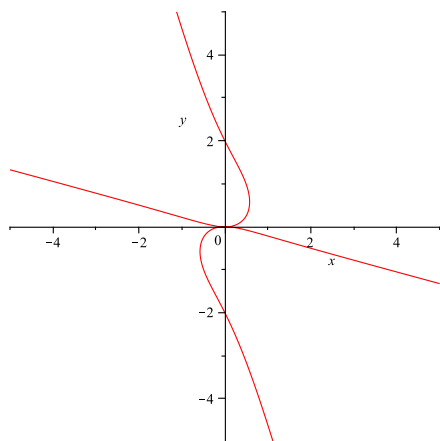

Another way to describe or define a curve in two dimensions is by way of *parametric equations*, $x = f(t)$ and $y = g(t)$, where the x and y coordinates of points on the curve are simultaneously specified by plugging a third variable, called the *parameter* (in this case t), into functions $f(t)$ and $g(t)$. This approach can come in handy for situations where it is impossible to describe all of a curve as the graph of a function of x (or of y) and arises pretty naturally in various physics problems. (Think of specifying, say, the position (x, y) of a moving particle at time t.)
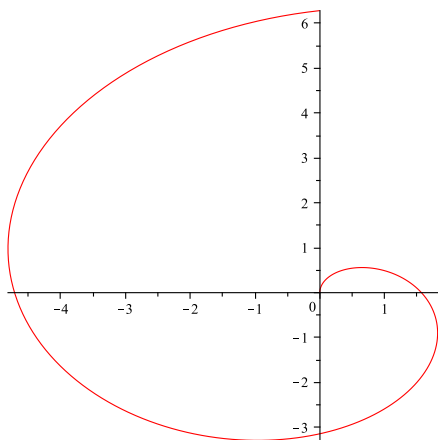
3. Use `Maple` to plot the parametric curves given by $x = \cos(t)$ and $y = \sin(t)$ for $0 \le t \le \pi$, $x = t\sin(t)$ and $y = t\cos(t)$ for $0 \le t \le 2\pi$, and $x = 2(1 - \cos(t))\cos(t)$ and $y = (1 - \cos(t))\sin(t)$ for $0 \le t \le 2\pi$. [Please submit a printout of your worksheet(s).] *[2]*

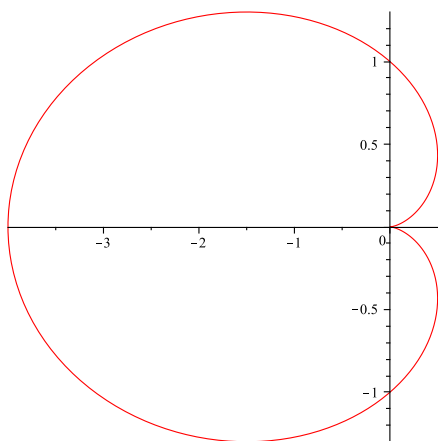SOLUTION. Parametric plots can be done with the basic plot command:.

```
> plot([cos(t), sin(t), t = 0..Pi])
```
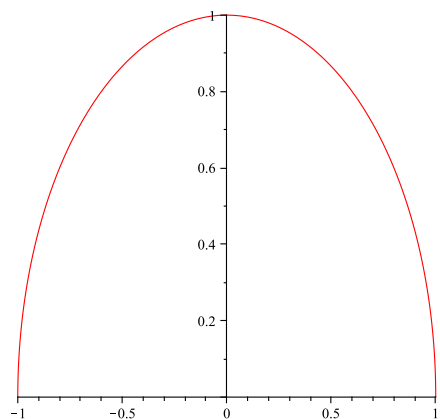
```
> plot([t*sin(t), t*cos(t), t = 0..2*Pi])
```



```
> plot([(2*(1-cos(t)))*cos(t), (1-cos(t))*sin(t), t = 0..2*Pi])
```


Polar coordinates are an alternative to the usual two-dimensional Cartesian coordinates. The idea is to locate a point by its distance r from the origin and its direction, which is given by the (counterclockwise) angle θ between the positive x-axis and the line from the origin to the point. Thus, if (r, θ) are the polar coordinates of some point, then its Cartesian coordinates are given by $x = r\cos(\theta)$ and $y = r\sin(\theta)$. (Note that for purposes of calculus it is usually more convenient to measure angles in radians rather than degrees.) Polar coordinates come in particularly handy when dealing with curves that wind around the origin, since such curves can often be conveniently represented by an equation of the form $r = f(\theta)$ for some function f of θ. If r is negative for a given θ, we interpret that as a distance of $|r|$ in the *opposite* direction, *i.e.* the direction $\theta + \pi$.

4. Use `Maple` to plot the curves in polar coordinates given by $r = 1$ for $0 \leq \theta \leq \pi$, $r = 2(1 - \cos(\theta))$ for $0 \leq \theta \leq 2\pi$, and $r = 1 + \cos(\theta)$ for $0 \leq \theta \leq 2\pi$, respectively. [Please submit a printout of your worksheet(s).] *[2]*

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SOLUTION. While it is possible to use polar coordinates in the regular `plot` command using the `coords = polar` option, it is usually preferable to use the `polarplot` command, which requires the `plot` package.
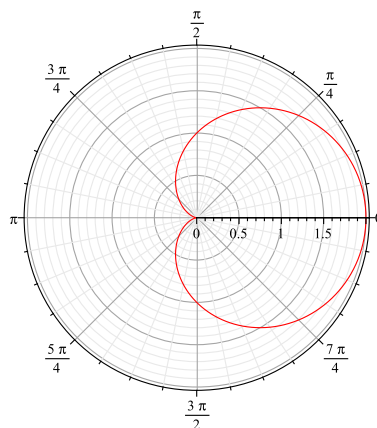
```
> polarplot(1, theta = 0..Pi)
```
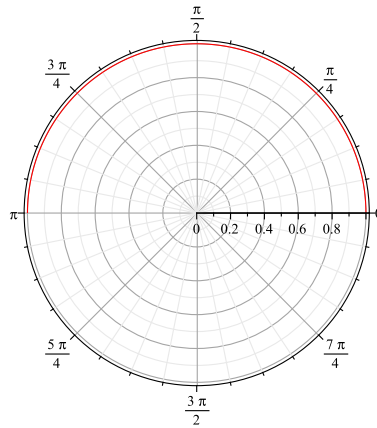


```
> polarplot(2*(1-cos(theta)), theta = 0..2*Pi)
```
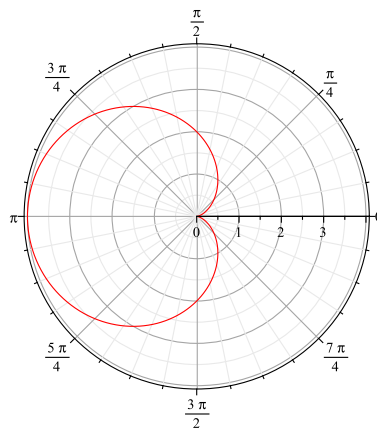


```
> polarplot(1+cos(theta), theta = 0..2*Pi)
```


□

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5. Some of the curves in problems **1**–**4** are actually the same curve. (With different presentations!) Which ones are the same? *[2]*

SOLUTION. Four curves, $y = \sqrt{1 - x^2}$ for $-1 \leq x \leq 1$ from **1**, $x^2 + y^2 = 1$ for $y \geq 0$ from **2**, $x = \cos(t)$ and $y = \sin(t)$ for $0 \leq t \leq \pi$ from **3**, and $r = 1$ for $0 \leq \theta \leq \pi$ in polar coordinates from **4**, are all the same curve, namely the upper unit semicircle with centre at the origin. Note that the default scaling used by `Maple` in **1**–**3** makes in look like half of an ellipse, rather than half of a circle.

None of the other curves are the same as any of the others, though there are some near misses. The last two curves in **4**, for example, are mirror images of one another. Neither is the same as the similar-looking curve in **3**, though if that had an extra factor of 2 in its y-coordinate function, it would be the same as the second curve in **4**. Finally, the third curve in **1** is the upper half of the second curve in **2**. ∎

REFERENCES

1. *A very quick start with* `Maple`, by Stefan Bilaniuk, which can be found (pdf) at:
   `euclid.trentu.ca/math/sb/1110H/Summer-2018/MATH1110H-maple-start.pdf`

2. *Formula for Success*, by Ruth Brandow, Ellen Dempsey, Marj Tellis, and Lisa Davies, Academic Skills Centre, Trent University, which can be found (pdf) at:
   `www.trentu.ca/academicskills/documents/ASC_mathematics_000.pdf`

3. *Single Variable Calculus* (Early Transcendentals), by David Guichard, licensed under the Creative Commons Attribution-NonCommercial-ShareAlike License. May be downloaded for free from: `communitycalculus.org`

4. *Getting started with Maple 10*, by Gilberto E. Urroz (2005), which can be found (pdf) at: `euclid.trentu.ca/math/sb/1110H/Summer-2018/GettingStartedMaple10.pdf`