Take-Home Quiz
Begging the question?
Due on Wednesday, 13 July, 2011.

We missed a quiz a few weeks ago because of a rescheduled appointment. This is a make-up to ensure that we have the minimum number of quizzes demanded by the marking scheme. The following problem is an oldie – it's over a century old! – but goodie:

Five beggars sat down in a circle, and each piled up, in a heap before him, the pennies he had received that day: and the five heaps were equal.

Then spake the eldest and wisest of them, unfolding, as he spake, an empty sack.

"My friends, let me teach you a pretty little game! First, I name myself 'Number One', my left-hand neighbour 'Number Two,' and so on to 'Number Five.' I then pour into this sack the whole of my earnings for the day, and hand it on to him who sits next but one on my left, that is 'Number Three.' *His* part in the game is to take out of it and give to his two neighbours, so many pennies as represent their names (that is, he must give four to 'Number Four' and two to 'Number Two'); he must then put *into* the sack half as much as it ccontained when he received it; and he must then hand it on just as I did, that is, he must hand it to him who sits next but one on his left—who will of course be 'Number Five.' *He* must proceed in the same way, and hand it on to 'Number Two,' from whom the sack will find its way to 'Number Four,' and so to me again. If any player cannot furnish, from his own heap, the whole of what he has to put into the sack, he is at liberty to draw upon any of the other heaps, *except mine*!"

The other beggars entered into the game with much enthusiasm: and in due time the sack returned to 'Number One,' who put into it the two pennies he had received during the game, and carefully tied up the mouth of it with a string. Then, remarking "it is a *very* pretty little game," he rose to his feet, and hastily quitted the spot. The other four beggars gazed at each other with rueful countenances. Not one of them had a penny left!

How much had each at first?

1. Solve the problem. Please explain your reasoning! *[5]*

SOLUTION. Here is the author's own solution:

Let x be the number of pennies each had at first.

No. (3) received x, took out $(2+4)$, and put in $\frac{x}{2}$; so that the sack then contained $x \cdot \frac{3}{2} - 6$. Let us write 'a' for '$\frac{3}{2}$'.

No. (5) received $xa - 6$, took out $(4+1)$, and put in enough to multiply, by a, its contents when he received it. The sack now contained $(xa^2 - 6a - 5)$.

No. (2) took out $(1+3)$, and handed on $(xa^3 - 6a^2 - 5a - 4)$.

No. (4) took out $(3+5$, and handed on $(xa^4 - 6a^3 - 5a^2 - 4a - 8)$.

No. (1) put in 2. The sack now contained $5x$.

Hence $xa^4 - 6a^3 - 5a^2 - 4a - 6 = 5x$;

$$\Leftrightarrow x = \frac{6a^3 + 5a^2 + 4a + 6}{a^4 - 5}$$

$$= \frac{6 \cdot 3^3 + 5 \cdot 3^2 \cdot 2 + 4 \cdot 3 \cdot 2^2 + 6 \cdot 2^3) \cdot 2}{3^4 - 5 \cdot 2^4}$$

$$= \frac{(162 + 90 + 48 + 48) \cdot 2}{81 - 80} = 696 = \textit{2l. 18s. 0d.}$$

Q.E.F.

It's obvious calculators weren't that common then . . . One question: What's with the "Q.E.F." at the end? ("Q.E.D." you've probably heard of.) □

Bonus: Who devised the problem above? *[1]*

SOLUTION. Charles Lutwidge Dodgson, better known under his pen name of Lewis Carroll, devised the problem. (It's problem number 52 from his book *Pillow Problems*.) These days he is remembered mainly for two books for children (and lots of adults!), *Alice in Wonderland* and *Through the Looking Glass*, and some nonsense verse, especially *Jabberwocky* and *The Hunting of the Snark*. In addition to writing stories and poems, he was a mathematician, an Anglican minister, and a photographer. His works include quite a bit of recreational mathematics. □