Mathematics 1100Y – Calculus I: Calculus of one variable
TRENT UNIVERSITY, SUMMER 2011

Assignment #7
An integral inequality
Due on Monday, 27 June, 2011.

Up / Down side: No `Maple`; it won't help. Down / Up side: It's a proof. (Well, a generic calculation or two, anyway.)

1. Suppose that $f(x)$ and $g(x)$ are continuous functions which are not always equal to 0 on some interval $[a, b]$. Show that

$$\left(\int_a^b f(x)g(x)\,dx\right)^2 \leq \left(\int_a^b f^2(x)\,dx\right)\left(\int_a^b g^2(x)\,dx\right) . \textit{ [10]}$$

NOTE: To do this you will probably want to review some of the basic properties of definite integrals, especially the order properties, given in Chapter 5 of the textbook.

Hint: Consider the case where there is some constant c such that $f(x) = cg(x)$ for all x in $[a, b]$ separately from the case where there is no such constant.

Bonus: A two-player game (in which the players take turns making moves) is considered to be finite if it cannot go on forever when played by the rules. The two-player game SUPERGAME is played as follows: the first player chooses a finite two-player game, which the two players proceed to play out with the second player going first. Is SUPERGAME itself a finite two-player game? Why or why not? *[1]*

Problems

Problems worthy
of attack
prove their worth
by hitting back.

Piet Hein