Mathematics 1100Y – Calculus I: Calculus of one variable
TRENT UNIVERSITY, SUMMER 2011

Assignment #4
Inverse hyperbolic trig functions
and an excess of fine print!
Due on Monday, 6 June, 2011.

The *hyperbolic trigonometric functions,* often just the *hyperbolic functions,* are so named because they relate angles to side length in triangles in the hyperbolic plane*, just as the ordinary trigonometric functions do in triangles in the Euclidean plane. The three principal ones are:

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

There are connections between the hyperbolic and the regular trigonometric functions, some of which will become apparent when we study series. Your main task in this assignment will be to work out the inverses of $\sinh(x)$ and $\cosh(x)$.

1. Use `Maple` to graph $\sinh(x)$ and $\cosh(x)$. *[1]*

2. Use your graphs from **1** to determine just how much of each of $\sinh(x)$ and $\cosh(x)$ could be inverted. *[2]*

3. Use `Maple`'s ability to solve equations symbolically to find expressions for the inverses of $\sinh(x)$ and $\cosh(x)$, namely $\operatorname{arcsinh}(x)$ and $\operatorname{arccosh}(x)$†. *[3]*

Note: The basic tool you will need to do **3** is `Maple`'s `solve` command, which has many options and variations. Be warned that while `Maple` is an enormously powerful and flexible tool for doing algebra symbolically, it suffers from a defect of this virtue: it is sometimes very hard for a non-expert user to figure out how to extract a useful result even for a relatively simple problem. Part of **3** boils down to solving a quadratic equation for e^x, and `Maple`'s obsession with generality makes this best done a little indirectly. *Make sure to ask for help if you run into trouble!*

4. Derive expressions for $\operatorname{arcsinh}(x)$ and $\operatorname{arccosh}(x)$ yourself. (If these are different from what `Maple` gave you for **3**, you may well be correct, but try to explain, if you can, why they amount to the same thing.) *[2]*

5. Find the derivatives of $\sinh(x)$, $\cosh(x)$, $\operatorname{arcsinh}(x)$, and $\operatorname{arccosh}(x)$. *[2]*

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* The hyperbolic plane is just like the Euclidean plane except that parallel lines work differently: instead of having just one line parallel to a given line through any point not on the given line, there are infinitely many lines parallel to the given line through any point not on the given line. (One immediate consequence is that the lines through a point parallel to a given line are not parallel to each other.) Hyperbolic geometry, and other non-Euclidean geometries, actually have uses. For one example, the key idea in general relativity is that mass and energy affect the curvature of space, giving it a non-Euclidean geometry.

† Our textbook uses the notation $\sinh^{-1}(x)$ and $\cosh^{-1}(x)$, respectively, for the inverses of $\sinh(x)$ and $\cosh(x)$.