Mathematics 1100Y – Calculus I: Calculus of one variable
Trent University, Summer 2011
Assignment #12
Are you series?
Due on Wednesday, 3 August, 2011.

This is an extra assignment which can be used to increase the pool from which the best ten are chosen to count toward your final mark. Doing it will also give you a little bit more practice with power and Taylor series.

1. Suppose x is a variable and a_n for $n \geq 0$ are constants such that

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$= \left(1 + x + x^2 + x^3 + \cdots\right)^2 = \left(\sum_{n=0}^{\infty} x^n\right)^2.$$

Find a formula for a_n in terms of n. *[4]*

HINT: Work out the first few a_ns by multiplying out $\left(1 + x + x^2 + x^3 + \cdots\right)^2$ and then collecting like terms, and look for a pattern.

2. Using the fact that $\arctan(x) = \int_0^x \frac{1}{1+t^2}\, dt$, find a formula for b_n in terms of n such that $\arctan(x) = \sum_{n=0}^{\infty} b_n x^n$. *[4]*

HINT: Express $\frac{1}{1+t^2}$ as a power series and then integrate it term-by-term.

3. Use your answer to **3** to find a series $\sum_{n=0}^{\infty} c_n$ whose sum is $\frac{\pi}{4}$. *[2]*

NOTE: Variations on this series have been used in the past to compute approximations to π. Nowadays people usually do so with other series that converge faster.