Mathematics 1100Y – Calculus I: Calculus of one variable

TRENT UNIVERSITY, Summer 2010

Test 1

7 June, 2010

Instructions

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator and an aid sheet.

1. Do any *two* (2) of **a**–**c**. *[10 = 2 × 5 each]*

a. Find the slope of the tangent line to $y = \tan(x)$ at $x = 0$.

b. Use the limit definition of the derivative to compute $f'(1)$ for $f(x) = x^2$.

c. Use the $\varepsilon - \delta$ definition of limits to verify that $\lim_{x \to 1}(2x - 1) = 1$.

2. Find $\frac{dy}{dx}$ in any *three* (3) of **a**–**d**. *[9 = 3 × 3 each]*

a. $y = \dfrac{x}{x+1}$ **b.** $x^2 + y^2 = 4$ **c.** $y = \displaystyle\int_0^x t\cos(3t)\,dt$ **d.** $y = \ln\left(x^3\right)$

3. Do any *two* (2) of **a**–**c**. *[10 = 2 × 5 each]*

a. Explain why $\lim_{x \to 0} \dfrac{x}{|x|}$ doesn't exist.

b. A spherical balloon is being inflated at a rate of 1 m^3/s. How is its radius changing at the instant that it is equal to 2 m? [The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.]

c. Use the Left-Hand Rule to find $\displaystyle\int_1^3 x\,dx$. $\left[\ \displaystyle\sum_{i=0}^{n-1} i = 0 + 1 + \cdots + (n-1) = \tfrac{n(n-1)}{2}\ \right]$

4. Let $f(x) = \dfrac{x^2}{x^2+1}$. Find the domain and all the intercepts, vertical and horizontal asymptotes, and maxima and minima of $f(x)$, and sketch its graph using this information. *[11]*

[Total = 40]

Bonus. Find any inflection points of $f(x) = \dfrac{x^2}{x^2+1}$ as well. *[3]*