

Mathematics 1100Y – Calculus I: Calculus of one variable

TRENT UNIVERSITY, Summer 2010

**Solution to Assignment #8
The Island of Knights and Knaves**

1. Every native of the Island of Knights and Knaves is either a knight who always tells the truth or a knave who always lies. While visiting the Island you meet six natives, whom we'll designate as A through F, who tell you the following:

A: Both D and E are knaves.

B: E is a knave or F is a knave.

C: E is a knight and F is a knight.

D: C could say that B is a knight.

E: D is a knave.

F: C could say that B is a knave.

Determine, as best you can, which of the six are knights and which are knaves[†]. [10]

SOLUTION. We will interpret “or” inclusively – as is normally the case in mathematical English – in this solution. That is, the statement “ α or β ” is true if at least one of statements α and β is true. Thus “ α or β ” is false only when both α and β are false. Here goes:

Note that B’s statement, “E is a knave or F is a knave,” is true exactly when C’s statement, “E is a knight and F is a knight,” is false and *vice versa*. Thus one of B and C must be telling the truth and the other lying, so one of B and C must be a knight and the other a knave.

Either way, D’s statement, “C could say that B is a knight,” must be false: if C is a knave and B is a knight, C must lie about it and say B is a knave, while if C is a knight and B is a knave, C must tell the truth and say B is a knave. Since D tells a falsehood, D must be a knave.

Similarly, F’s statement, “C could say that B is a knave,” must be true: if C is a knave and B is a knight, C must lie about it and say B is a knave, while if C is a knight and B is a knave, C must tell the truth and say B is a knave. Since F tells the truth, F must be a knight.

Since D is a knave, E’s statement, “D is a knave,” is truthful, and hence E is a knight. It then follows that A’s statement, “Both D and E are knaves,” is a falsehood, so A is a knave.

Since E is a knight and F is a knight, B’s statement, “E is a knave or F is a knave,” is false, and C’s statement, “E is a knight and F is a knight,” is true. Thus B is a knave and C is a knight.

Hence A, B, and D are knaves and C, E, and F are knights. ■

[†] This puzzle was adapted from one by Zachary Ernst