Mathematics 1100Y – Calculus I: Calculus of one variable
Trent University, Summer 2010
Final Examination

Time: *09:00–12:00, on Friday, 30 July, 2010.* *Brought to you by* Стефан Біланюк.
Instructions: Show all your work and justify all your answers. *If in doubt,* **ask!**
Aids: Calculator; one aid sheet (all sides!); one brain (no limit on active neurons).

Part I. Do all three (3) of **1–3**.

1. Compute $\frac{dy}{dx}$ as best you can in any *three* (3) of **a–f**. *[15 = 3 × 5 each]*

a. $x^2 + 3xy + y^2 = 23$ **b.** $y = \ln(\tan(x))$ **c.** $y = \int_x^3 \ln(\tan(t))\, dt$

d. $y = \frac{e^x}{e^x - e^{-x}}$ **e.** $\begin{matrix} x = \cos(2t) \\ y = \sin(3t) \end{matrix}$ **f.** $y = (x+2)e^x$

2. Evaluate any *three* (3) of the integrals **a–f**. *[15 = 3 × 5 each]*

a. $\int_{-\pi/4}^{\pi/4} \tan(x)\, dx$ **b.** $\int \frac{1}{t^2 - 1}\, dt$ **c.** $\int_0^{\pi} x\cos(x)\, dx$

d. $\int \sqrt{w^2 + 9}\, dw$ **e.** $\int_1^e \ln(x)\, dx$ **f.** $\int \frac{e^x}{e^{2x} + 2e^x + 1}\, dx$

3. Do any *five* (5) of **a–i**. *[25 = 5 × 5 ea.]*

a. Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $0 \le x \le 4$, the x-axis, and $x = 4$, about the x-axis.

b. Use the $\varepsilon - \delta$ definition of limits to verify that $\lim_{x \to 1} 3x = 3$.

c. Find the Taylor series of $f(x) = \frac{x^2}{1 - x^2}$ at $a = 0$ without taking any derivatives.

d. Sketch the polar curve $r = 1 + \sin(\theta)$ for $0 \le \theta \le 2\pi$.

e. Use the limit definition of the derivative to compute $f'(1)$ for $f(x) = x^2$.

f. Use the Right-hand Rule to compute the definite integral $\int_1^2 \frac{x}{2}\, dx$.

g. Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$ converges absolutely, converges conditionally, or diverges.

h. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n^2}{\pi^n} x^n$.

i. Compute the arc-length of the polar curve $r = \theta$, $0 \le \theta \le 1$.

Part II. Do any *two* (2) of **4–6**.

4. Find the domain, all maximum, minimum, and inflection points, and all vertical and horizontal asymptotes of $f(x) = e^{-x^2}$, and sketch its graph. *[15]*

5. Find the area of the surface obtained by rotating the curve $y = \tan(x)$, $0 \le x \le \frac{\pi}{4}$, about the x-axis. *[15]*

6. Find the volume of the solid obtained by rotating the region below $y = 1 - x^2$, $-1 \le x \le 1$, and above the x-axis about the line $x = 2$. *[15]*

Part III. Do *one* (1) of **7** or **8**.

7. Do all three (3) of **a–c**.

a. Use Taylor's formula to find the Taylor series of e^x centred at $a = -1$. *[7]*

b. Determine the radius and interval of convergence of this Taylor series. *[4]*

c. Find the Taylor series of e^x centred at $a = -1$ using the fact that the Taylor series of e^x centred at 0 is $\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$. *[4]*

8. Do all three (3) of **a–c**. You may assume that the Taylor series of $f(x) = \ln(1+x)$ centred at $a = 0$ is $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \cdots$.

a. Find the radius and interval of convergence of this Taylor series. *[6]*

b. Use this series to show that $\ln\left(\frac{3}{2}\right) = \frac{1}{2} - \frac{1}{8} + \frac{1}{24} - \frac{1}{64} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n2^n}$. *[3]*

c. Find an n such that $T_n\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{1}{8} + \frac{1}{24} - \frac{1}{64} + \cdots + \frac{(-1)^{n+1}}{n2^n}$ is guaranteed to be within $0.01 = \frac{1}{100}$ of $\ln\left(\frac{3}{2}\right)$. *[6]*

[Total = 100]

Part IV - Something different. Bonus!

$\mathbf{e^{i\pi}}$**.** Write a haiku touching on caclulus or mathmatics in general. *[2]*

haiku?

seventeen in three:
five and seven and five of
syllables in lines

I HOPE THAT YOU ENJOYED THE COURSE. ENJOY THE REST OF THE SUMMER!