Mathematics 1110H (Section A) – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2024

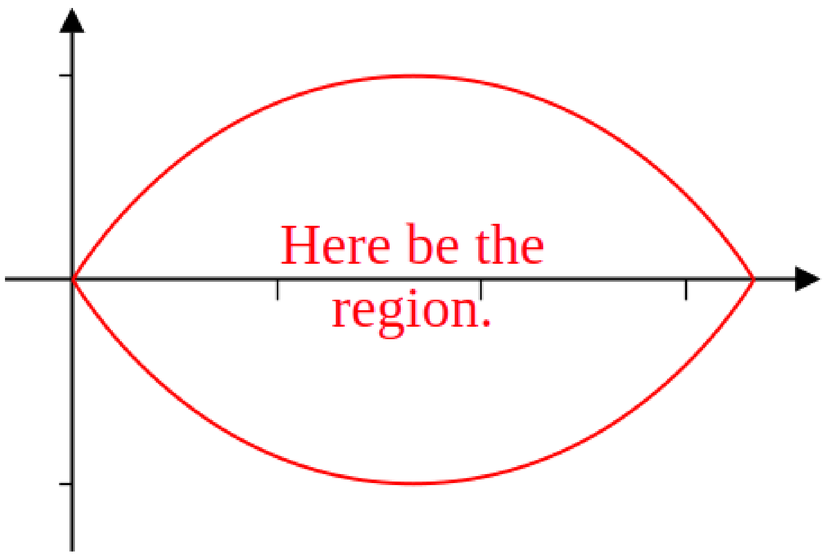
Solutions to Quiz #10
Crank up the volume!

No restrictions on the methods you are allowed to use to solve the following problems, but whichever you choose, please show all your work!

Consider the region between $y = \sin(x)$ and $y = -\sin(x)$, $0 \le x \le \pi$, and the solid obtained by revolving about the line $y = -1$.

1. Sketch the region. *[0.5]*

SOLUTION. Here is a sketch:



$\square$

2. Find the area of the region. *[1]*

SOLUTION 1. *By hand.* Here we go:

$$\begin{aligned} \text{Area} &= \int_0^\pi (\sin(x) - [-\sin(x)])\,dx = \int_0^\pi 2\sin(x)\,dx = -2\cos(x)\Big|_0^\pi \\ &= -2\cos(\pi) - (-2\cos(0)) = -2(-1) - (-2\cdot 1) = 2 + 2 = 4 \quad \square \end{aligned}$$

SOLUTION 2. *Using SageMath.* Same setup as above, but with SageMath evaluating the integral:

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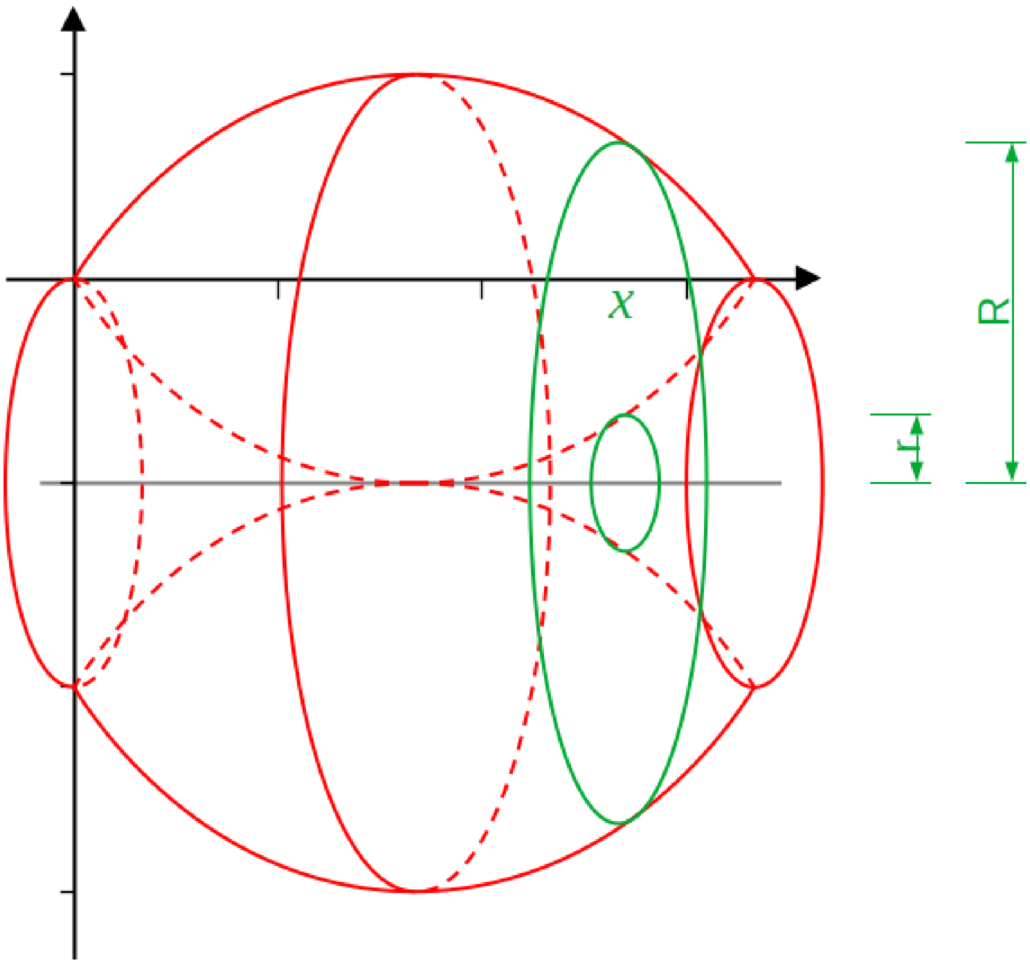
[1]: integral( sin(x) - (-sin(x)), x, 0, pi )
[1]: 4
```

$\square$

3. Sketch the solid. *[0.5]*

SOLUTION. Here is a sketch with a washer cross-section drawn in:



$\square$

4. Find the volume of the solid. *[3]*

SOLUTION 1. *By hand, with the disk/washer method.* Consider the "washer" cross-section of the solid in the diagram for question **3**. If this is the cross-section at x, it has outer radius $R = \sin(x) - (-1) = 1 + \sin(x)$ and inner radius $r = -\sin(x) - (-1) = 1 - \sin(x)$, and hence area $A(x) = \pi R^2 - \pi r^2 = \pi \left((1 + \sin(x))^2 - (1 - \sin(x))^2 \right)$. The volume of the solid is then:

$$\begin{aligned} V &= \int_0^\pi A(x)\,dx = \int_0^\pi \pi \left((1+\sin(x))^2 - (1-\sin(x))^2 \right)\,dx \\ &= \pi \int_0^\pi \left(\left(1 + 2\sin(x) + \sin^2(x)\right) - \left(1 - 2\sin(x) + \sin^2(x)\right) \right)\,dx = \pi \int_0^\pi 4\sin(x)\,dx \\ &= -4\pi\cos(x)\big|_0^\pi = (-4)\pi(-1) - (-4)\pi 1 = 4\pi + 4\pi = 8\pi \quad \square \end{aligned}$$

SOLUTION 2. *Using SageMath, with the disk/washer method.* As above, with the integration done by SageMath:

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[2]: integral( pi*((1+sin(x))^2 - (1-sin(x))^2), x, 0, pi )

[2]: 8*pi
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■

NOTE. It's possible to find the volume with the cylindrical shell method, but the integrals involve arcsin and are rather more difficult.