Mathematics 1110H (Section A) – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2024
Assignment #2
Sagely Analysis
*Due on Friday, 4 October.**

1. Consider the quadratic function $f(x) = x^2 - 5x + 6$.

    a. Use SageMath to plot $y = f(x)$ for $0 \le x \le 5$. Use the graph to guess at the roots of $y = f(x)$. *[1]*

    b. Use SageMath to find the roots of $y = f(x)$ by solving the equation $f(x) = 0$. *[1]*

    c. Use the quadratic formula by hand to find the roots of $y = f(x)$. *[0.5]*

2. Consider the quartic function $g(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$.

    a. Use SageMath to plot $y = g(x)$ for $0 \le x \le 5$. Use the graph to guess at the roots of $y = g(x)$. *[1]*

    b. Use SageMath to find the roots of $y = g(x)$ by solving the equation $g(x) = 0$. *[1]*

    c. What polynomial is the function $h(x) = g(x)/f(x)$ equal to, except, of course, when $f(x) = 0$? *[0.5]*

    d. Use the quartic formula by hand to find the roots of $y = g(x)$. *[Bonus = 1]*

NOTE. The bonus mark for part **d** will be hard-earned if you choose to try ...

3. **a.** Use SageMath to plot $y = x$ and $y = \sin(x)$ together for $-\pi \le x \le \pi$. *[1]*

    b. Use SageMath to plot $y = |x|$ and $y = |\sin(x)|$ together for $-\pi \le x \le \pi$. *[1]*

    c. Use plots drawn by SageMath to make an argument that $|\sin(x)| \le |x|$ for all x, and that equality occurs only when $x = 0$. *[1]*

NOTE. For part **c**, it might help to draw some additional plots for other ranges of x.

4. Use SageMath to find the inverse function of $s(x) = \dfrac{e^x - e^{-x}}{2}$. *[2]*

Hint: $t(x)$ is the inverse function of $s(x)$ if $y = t(x)$ exactly when $x = s(y)$. Also, ask yourself whether the answers SageMath gave you make sense.