

Mathematics 1110H (Section B) – Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Fall 2023

Solutions to Mini-Assignment Quiz #1

Jack and Jill Hiked on the Hill

NOTE. The weekly quizzes will normally be based on the material covered in lecture in the previous week. This first quiz therefore ought to be based on the contents of a single lecture, part of which had to do with the organization of the course rather than its content. That's too little to test, so this quiz instead asks you to solve an arithmetical puzzle. Have fun!

Don't forget that while you are allowed to work together and look things up, what you submit should be written up entirely by yourself, and give due credit to any collaborators or sources that you ended up actually using.

Jack and Jill go hiking on a trail on the crest of a ridge that has a mix of flat and sloped ground. They leave their starting point at 9:00 a.m., hike to a point on the ridge, have a half-hour lunch there, and then return by exactly the same path they followed going out (in reverse, of course :-), coming back to the starting point at 3:30 p.m. on the same day. For simplicity in answering the questions below, we will assume that Jill and Jack always walk at 2 km/h on level ground, 1.25 km/h going uphill, and 5 km/h going downhill.

1. How many kilometers, along the trail followed by Jill and Jack, is it from the starting point to the point where they had lunch? [2.5]

SOLUTION. A level kilometre takes $\frac{1}{2}$ of an hour for the hikers, a kilometre uphill takes $\frac{1}{1.25} = \frac{4}{5}$ of an hour, and a kilometre downhill takes $\frac{1}{5}$ of an hour. Hence to go and return over the same kilometre, whether on the level or on sloped ground, takes an hour, since $\frac{1}{2} + \frac{1}{2} = \frac{4}{5} + \frac{1}{5} = 1$. Hence in 6 hours of hiking – the half-hour lunch break doesn't count here – they went 6 kilometres out and 6 back. Thus the point where Jack and Jill had lunch is 6 km from their starting point. \square

2. Determine, as accurately as you can, when Jack and Jill had lunch. [2.5]

SOLUTION. If the 6 kilometres they hiked to get to the lunch point had been nearly all level, Jill and Jack would have taken about $\frac{6}{2} = 3$ hours to get there; if it had been almost all uphill, a bit under $\frac{6}{1.25} = 4.8$ hours; and if it had been almost all down hill, a bit more than $\frac{6}{5} = 1.2$ hours. It follows that Jill and Jack started their half-hour lunch somewhere between 10:12 a.m. and 1:48 p.m. \blacksquare

NOTE. This problem was ~~stolen from~~ inspired by *Knot I* of Lewis Carroll's[†] *A Tangled Tale*, although the details of the setup have been modified.

[†] The pen name of Charles Lutwidge Dodgson (1832-1898), best known nowadays as the author of *Alice in Wonderland*. He was a mathematician, poet, and photographer, as well as an author.