Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2023

Assignment #2
Solving equations with SageMath
Due just before midnight on Friday, 6 October.*

If you want to get started on this assignment before attending your lab in MATH 1110H, skimming and later referring to as necessary to Sections 1.4.2, 1.8, and 4.8 of Gregory Bard's book *Sage for Undergraduates* (in the SageMath folder in the Course Content section on Blackboard), and perhaps also Appendix G of this book (which is in a separate file from the rest of the book), is probably going to be useful.

1. The Indian mathematician Bhaskara (1114-1185 A.D.), often called Bhaskara II to distinguish him from an earlier mathematician named Bhaskara (*c.* 600-680 A.D.), posed the following problem in a book dedicated to his daughter Lilavati:

> *Seven times half the square root of a flock of geese was observed to march slowly away and two were seen fighting playfully in the water. Say, what was the number of geese?*

a. Restate the problem given above as an equation. *[1]*

b. Solve the equation you obtained in **a** by hand. Show all your work! *[1]*

c. Solve the equation you obtained in **a** using SageMath. [*Hint:* `solve`] *[1]*

2. The *hyperbolic functions* include:

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$
$$\operatorname{csch}(x) = \frac{1}{\sinh(x)} \qquad \operatorname{sech}(x) = \frac{1}{\cosh(x)} \qquad \coth(x) = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

The names of these function are usually pronounced something like "sinch", "kosh", "tanch", "co-seech", "seech", and "kotch", respectively. They turn out to be closely related to the natural exponential function (obviously) and the trigonometric functions; the latter connections being more obvious when you look at their series expansions and especially when you start looking at them as functions of a complex variable.

a. Explain why $-1 < \tanh(x) < 1$ for all x. *[1]*

b. Use SageMath to compute $\lim_{x\to-\infty} \tanh(x)$ and $\lim_{x\to\infty} \tanh(x)$. [*Hint:* `lim`] *[1]*

c. What does the information in **a** and **b** tell you about $y = \tanh(x)$? What does it tell you about the inverse function, $y = \operatorname{arctanh}(x)$, of $\tanh(x)$? *[1]*

d. Find a formula for $\operatorname{arctanh}(x)$ by hand. Show all your work. *[1.5]*

e. Find a formula for $\operatorname{arctanh}(x)$ using SageMath. [*Hint:* `solve`] *[1.5]*

f. Use SageMath to graph both $y = \tanh(x)$ and $y = \operatorname{arctanh}(x)$ in the same plot, for $-2.5 \le x \le 2.5$ and $-2.5 \le y \le 2.5$. [*Hint:* Add the plots. Literally.] *[1]*

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* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper.