Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2021
Take-Home Final Examination

Available on Blackboard from 12:00 a.m. on Monday, 13 December.
Due on Blackboard by 11:59 p.m. on Wednesday, 15 December.

Submission: Scans or photos of handwritten work are entirely acceptable so long as they are legible and in some common format; solutions submitted as a single pdf are strongly preferred. If submission via Blackboard's Assignments module fails repeatedly, then, as a last resort, email them to the instructor at: `sbilaniuk@trentu.ca`

Allowed aids: For this exam, you are permitted to use your textbook and all other course material, including that on Blackboard and the archive page, from this and any other mathematics course(s) you have taken or are taking now, but *you may not use any other sources or aids, nor give or receive any help*, except to ask the instructor to clarify questions and to use a calculator (any that you like).

Instructions: Do parts **A** and **B**, and, if you wish, part **C**. Please show all your work and justify all your answers. *If in doubt about something,* **ask!**

Part A. Do *all four* (4) of **1**–**4**. *[Subtotal = 72]*

1. Compute $\dfrac{dy}{dx}$ as best you can in any *five* (5) of **a**–**f**. *[20 = 5 × 4 each]*

a. $\cos(x+y)=0$ **b.** $y=\left(x^2+1\right)^{13}$ **c.** $y=\displaystyle\int_{-\sin(x)}^{0} \arcsin(t)\,dt$

d. $y=e^{x(x-1)}$ **e.** $y=\dfrac{x+1}{x^2-1}$ **f.** $y=\left(x^2+1\right)\arctan(x)$

2. Evaluate any *five* (5) of the integrals **a**–**f**. *[20 = 5 × 4 each]*

a. $\displaystyle\int_1^e \ln\left(x^{17}\right)\,dx$ **b.** $\displaystyle\int_0^{\pi/8} \sec^3(2x)\,dx$ **c.** $\displaystyle\int_{-1}^{1} \frac{x^2-1}{x^4-1}\,dx$

d. $\displaystyle\int \frac{(\ln(x)+1)^2}{2x}\,dx$ **e.** $\displaystyle\int x\sec^2 x\,dx$ **f.** $\displaystyle\int x^x\cdot(\ln(x)+1)\,dx$

3. Do any *five* (5) of **a**–**i**. *[20 = 5 × 4 each]*

a. Find all the local maxima and minima, if any, of $y=x^4-18x^2$.

b. Sketch the region whose border consists of the curves $y=x^2$ for $0\le x\le 1$, $y=2-x$ for $1\le x\le 2$, and $y=-\sqrt{1-(x-1)^2}$ for $0\le x\le 2$, and find its area.

c. Use the ε–δ definition of limits to verify that $\lim\limits_{x\to 1}\sqrt{|x-1|}=0$.

d. Sketch the solid obtained by revolving the region between $y=\ln(x)$ and $y=0$, for $1\le x\le e$, about the x-axis, and find the volume of this solid.

More questions on page 2 . . .

and here they are:

e. Find any and all vertical and horizontal asymptotes of $y = \dfrac{1}{x+1} + \dfrac{1}{x-1}$.

f. Compute $\lim_{x \to \infty} x^3 e^{-x}$.

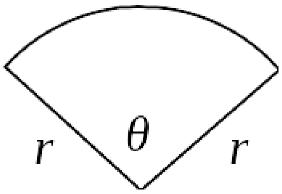
g. Use the limit definition of the derivative to show that $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right) = -\dfrac{2}{x^3}$.

h. A rectangular box is 1 m wide, x m long, and y m high. What is the minimum possible surface area of such a box if has a volume of 4 m^3?

i. Show that $\ln(\sec(x) - \tan(x)) = -\ln(\sec(x) + \tan(x))$.

4. Find the domain as well as any (and all) intercepts, vertical and horizontal asymptotes, intervals of increase, decrease and concavity, and maximum, minimum, and inflection points of $f(x) = e^{-1/x}$, and sketch its graph based on this information. *[12]*

Part B. Do any *two* (2) of **5**–**8**. *[Subtotal = 28 = 2 × 14 each]*

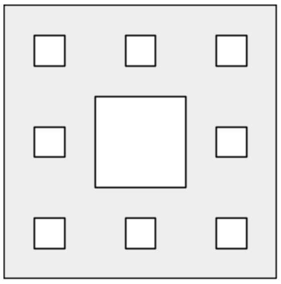


5. A pie slice with angle θ *rad* at the tip is cut out of a circle of radius r. What is the minimum possible perimeter of such a slice if it has an area of 16 cm^2?

6. A small stone is dropped into a still pool, creating a circular ripple that moves outward from the point of impact at a constant rate. How is the area enclosed by the ripple changing after 2 s if the circumference of the ripple is changing at a rate of 2π m/s at this instant?

7. Sketch the solid obtained by revolving the region between $y = \cos(x)$ and $y = \sin(x)$, for $\dfrac{\pi}{4} \leq x \leq \dfrac{5\pi}{4}$, about the y-axis and find its volume.

8. Suppose we start with a unit square. In step 1 we divide it into $3 \times 3 = 9$ subsquares and then remove the middle one. In step 2 we divide each of the eight remaining subsquares into $3 \times 3 = 9$ smaller subsquares and remove the middle one in each case. (The picture at right is what you have after step 2.) At each step $n \geq 3$, we subdivide the remaining subsquares into $3 \times 3 = 9$ even smaller subsquares and remove the middle one in each case. What is the area of the object remaining after infinitely many steps?



[Total = 100]

Part C. *Bonus problems!* If you feel like it, do one or both of these.

9. Recall that an integer greater than 1 is a prime number if it is not the product of two smaller positive integers. Determine whether or not the polynomial $p(x) = x^2 + x + 41$ always gives you prime numbers when $x \geq 0$ is an integer. *[1]*

10. Write an original poem touching on calculus or mathematics in general. *[1]*

I HOPE THAT YOU ENJOYED THE COURSE. HAVE A GOOD BREAK!