Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals (Section C)
TRENT UNIVERSITY, Fall 2021

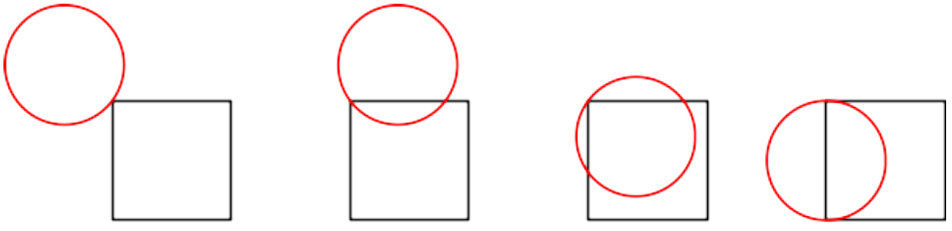
Assignment #5
Solving equations with SageMath
Due on Friday, 26 November.

Submission: Scanned or photographed handwritten solutions are fine, so long as they are legible. Submission as a single pdf is strongly preferred, but other common formats are probably OK. (If not, we'll get back to you! :-) Please submit via Blackboard's Assignments module. If that fails, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`

Before attempting the questions below, it would be useful to skim though Chapter 1 of *Sage for Undergraduates* by Gregory Bard, and keep it handy as a reference in case you need more than just Chapter 1. You can find a links to a pdf of this book, as well as some other resources, in the SageMath folder in the Course Content section of the MATH 1110H-C Blackboard site. Don't forget that while you may work together and look stuff up for the assignments. you should write and/or type up what you submit by yourself.

NOTE. *If you would rather use a comparable program other than* `SageMath`, *such as* `Maple` *or* `Mathematica`, *you may do so.*

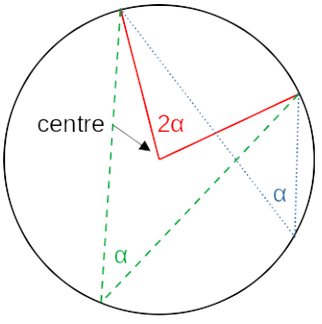
A circle of radius 1 is attached at a point on its circumference to one corner of a square with sides of length 2. The attachment allows the circle to be freely rotated about the point of attachment, making it possible to have the circle overlap the square in various ways:



It should be pretty obvious from the leftmost part of the diagram above that the minimum possible area of the overlap between the circle and the square is 0, since a single point doesn't cover much area. Your task in this assignment is to work out the maximum possible area of the overlap between the circle and the square.

1. Set this up as an optimization problem that you can use calculus to help solve. *[4]*

NOTE. This means, in particular, writing the area of the overlap in terms of a single variable, though you may need to divide up the problem into cases depending on just how the circle and the square overlap. You may also find the Central Angle Theorem – that a central angle in a circle is twice any corresponding inscribed angle – to be of use.



More questions on page 2! :-)

2. As best you can, solve the optimization problem you set up in answering question **1** by hand. *[3]*

3. As best you can, solve the optimization problem you set up in answering question **1** using `SageMath` to take derivatives and solve equations. *[3]*