Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals (Section C)
TRENT UNIVERSITY, Fall 2021

Assignment #4
Plotting with SageMath

Due on Friday, 12 November.

Submission: Scanned or photographed handwritten solutions are fine, so long as they are legible. Submission as a single pdf is strongly preferred, but other common formats are probably OK. (If not, we'll get back to you! :-) Please submit via Blackboard's Assignments module. If that fails, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`

Before attempting the questions below, please read through Chapter 1 of the text for the basics of graphing functions in Cartesian coordinates, and Chapter 10 for the basics of parametric curves and polar coordinates, respectively. Some basics of graphing in Cartesian coordinates, using trigonometric functions, and so on, can also be found in the Academic Skills pamphlet *Formula for Success*. The basic definitions of how parametric curves and polar coordinates work are given in this assignment for your convenience, but you might want some additional explanations and examples. You should also read the handout *Getting Started with* `sage.trentu.ca` and play around with `SageMath` a little. It might also be useful to skim though Chapter 1 of *Sage for Undergraduates* by Gregory Bard, and perhaps keep it handy as a reference. You can find links to these documents in the SageMath folder in the Course Content section of the MATH 1110H-C Blackboard site. Make use of each other and the instructor, too! Don't forget that while you may work together and look stuff up for the assignments. you should write and/or type up what you submit by yourself.

NOTE. *If you would rather use a comparable program other than* `SageMath`*, such as* `Maple` *or* `Mathematica`*, you may do so.*

A curve is easy to graph, at least in principle, if it can be described by a function of x in Cartesian coordinates.

1. Use `SageMath` to plot the curves defined by $y = x^2$, $y = 1 - x^2$, $y = \sqrt{x^2}$, and $y = \sqrt{1 - x^2}$, respectively, for $-1 \le x \le 1$ in each case. [Please submit a printout of the plots.] *[2]*

In many cases, a curve is difficult to break up into pieces that are defined by functions of x (or of y) and so is defined implicitly by an equation relating x and y; that is, the curve consists of all points (x, y) such that x and y satisfy the equation. One can, of course, also use implicit definitions to describe curves that can also be defined as the graphs of functions.

2. Use `SageMath` to plot the curves implicitly defined by $x^2+y^2 = 1$ for $y \ge 0$, $y^2-x^2 = 0$ for $-1 \le x \le 1$, and $\left(x^2 + y^2\right)^2 + 4xy\left(x^2 + y^2\right) - 4y^2 = 0$ for all x and y satisfying the equation, respectively. [Please submit a printout of the plots.] *[2]*

Another way to describe or define a curve in two dimensions is by way of *parametric equations,* $x = f(t)$ and $y = g(t)$, where the x and y coordinates of points on the curve are simultaneously specified by plugging a third variable, called the *parameter* (in this case t), into functions $f(t)$ and $g(t)$. This approach can come in handy for situations where it is impossible to describe all of a curve as the graph of a function of x (or of y) and arises pretty naturally in various physics problems. (Think of specifying, say, the position (x, y) of a moving particle at time t.)

3. Use `SageMath` to plot the parametric curves given by $x = \cos(t)$ and $y = \sin(t)$ for $0 \leq t \leq \pi$, $x = t\sin(t)$ and $y = t\cos(t)$ for $0 \leq t \leq 2\pi$, and $x = 2\,(1 - \cos(t))\cos(t)$ and $y = (1 - \cos(t))\sin(t)$ for $0 \leq t \leq 2\pi$. [Please submit a printout of the plots.] *[2]*

Polar coordinates are an alternative to the usual two-dimensional Cartesian coordinates. The idea is to locate a point by its distance r from the origin and its direction, which is given by the (counterclockwise) angle θ between the positive x-axis and the line from the origin to the point. Thus, if (r, θ) are the polar coordinates of some point, then its Cartesian coordinates are given by $x = r\cos(\theta)$ and $y = r\sin(\theta)$. (Note that for purposes of calculus it is usually more convenient to measure angles in radians rather than degrees.) Polar coordinates come in particularly handy when dealing with curves that wind around the origin, since such curves can often be conveniently represented by an equation of the form $r = f(\theta)$ for some function f of θ. If r is negative for a given θ, we interpret that as a distance of $|r|$ in the *opposite* direction, *i.e.* the direction $\theta + \pi$.

4. Use `SageMath` to plot the curves in polar coordinates given by $r = 1$ for $0 \leq \theta \leq \pi$, $r = 2\,(1 - \cos(\theta))$ for $0 \leq \theta \leq 2\pi$, and $r = 1 + \cos(\theta)$ for $0 \leq \theta \leq 2\pi$, respectively. [Please submit a printout of the plots.] *[2]*

5. Some of the curves in problems **1**–**4** are actually the same curve. (With different presentations!) Which ones are the same? *[2]*