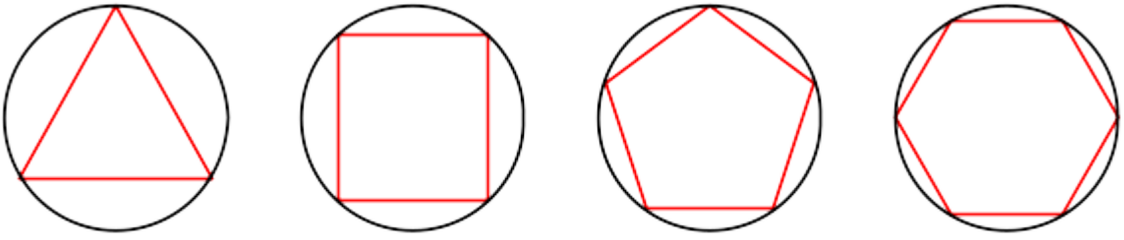
Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals (Section C)
TRENT UNIVERSITY, Fall 2021

Assignment #2
Limits of Perimeters

Due on Friday, 8 October.

Submission: Scanned or photographed handwritten solutions are fine, so long as they are legible. Submission as a single pdf is strongly preferred, but other common formats are probably OK. (If not, we'll get back to you! :-) Please submit via Blackboard's Assignments module. If that fails, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`

We can inscribe* a regular n-gon in a circle of radius 1 for any $n \geq 3$, as in the sketch below for $n = 3$ through $n = 6$.



Suppose we let $P(n)$ be the length of the perimeter of a regular n-gon inscribed in a circle of radius 1.

1. Give an intuitive explanation as to why $\lim_{n\to\infty} P(n) = 2\pi$. *[1]*

2. Show that $P(n) = 2n \sin\left(\frac{\pi}{n}\right)$. *[5]*

Hint: Connect the vertices of the polygon to the centre of the circle to make a bunch of isosceles triangles.

3. Use **1** and **2** to show that $\lim_{n\to\infty} \frac{n}{\pi} \sin\left(\frac{\pi}{n}\right) = 1$. *[2]*

4. Use **3** to give an argument that $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$. *[2]*

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* That is, draw the polygon so that its vertices are all on the circle.