Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals
TRENT UNIVERSITY, Fall 2020

Solutions to Assignment #4
Plotting with Cartesian and Polar Coordinates
Due on Friday, 13 November.

A curve is easy to graph, at least in principle, if it can be described by a function of x in Cartesian coordinates.

1. Use `Maple` to plot the curves defined by $y = 1$ for $-1 \le x \le 1$, $y = x^2$ for $-1 \le x \le 1$, $y = \sqrt{1 - x^2}$ for $-1 \le x \le 1$, and $y = -\frac{1}{3}\sqrt{9 - x^2}$ for $-3 \le x \le 3$. Describe each curve informally. [Please submit a pdf of your worksheet(s) if at all possible.] *[2]*

Hint: The first curve is the piece of the horizontal line $y = 1$ for $-1 \le x \le 1$. Really not much one can say about it ... :-)

SOLUTION. Please see the pdf of the `Maple` worksheet appended to the solutions for all the plots.

Note that the plots do not necessarily have x and y plotted to the same scale, which will distort the curves in many cases. That said, the first curve is the piece of the horizontal line $y = 1$ for $-1 \le x \le 1$; the second is the parabola $y = x^2$ for $-1 \le x \le 1$; the third is a semi-circle, being the upper half of the unit circle centred at the origin; and the fourth is a semi-ellipse, being the lower half of the ellipse with (horizontal) semi-major axis 3 and (vertical) semi-minor axis 1, also centred at the origin. ■

In many cases, a curve is difficult to break up into pieces that are defined by functions of x (or of y) and so is defined implicitly by an equation relating x and y; that is, the curve consists of all points (x, y) such that x and y satisfy the equation. One can, of course, also use implicit definitions to describe curves that can also be defined as the graphs of functions.

2. Use `Maple` to plot the curves implicitly defined by $x^2 + y^2 = 1$, $x^2 + 9y^2 = 9$, and $\left(x^2 + y^2\right)^2 + 4x\left(x^2 + y^2\right) - 4y^2 = 0$, each one for all those (x, y) for which the respective equation holds. Describe each these curves informally. [Please submit a pdf of your worksheet(s) if at all possible.] *[2]*

Hint: `Maple` has a dedicated `implicitplot` command that is part of the `plots` package.

SOLUTION. Please see the pdf of the `Maple` worksheet appended to the solutions for all the plots. Note the use of `with(plots)` to load the `plots` package before using the `implicitplot` and (for **4**) `polarplot` commands.

Again, note that the plots do not necessarily have x and y plotted to the same scale, which will distort the curves in many cases. That said, the first curve is the unit circle centred at the origin; the second is the ellipse with (horizontal) semi-major axis 3 and (vertical) semi-minor axis 1, also centred at the origin; and the third is a *cardioid* (*i.e.* heart-shaped curve) with x-intercepts at $x = -4$ and $x = 0$ and y-intercepts at $y = -2$, $y = 0$, and $y = 2$. ■

Another way to describe or define a curve in two dimensions is by way of *parametric equations,* $x = f(t)$ and $y = g(t)$, where the x and y coordinates of points on the curve are simultaneously specified by plugging a third variable, called the *parameter* (in this case t), into functions $f(t)$ and $g(t)$. This approach can come in handy for situations where it is impossible to describe all of a curve as the graph of a function of x (or of y) and arises pretty naturally in various physics problems. (Think of specifying, say, the position (x, y) of a moving particle at time t. This is pretty much the context in which Newton invented his version of calculus.)

3. Use `Maple` to plot the parametric curves given by $x = \cos(t)$ and $y = \sin(t)$ for $0 \leq t \leq 2\pi$, by $x = t\cos(t)$ and $y = t\sin(t)$ for $0 \leq t \leq 2\pi$, and $x = 3\cos(t)$ and $y = \sin(t)$ for $\pi \leq t \leq 2\pi$. Describe each of these curves informally. [Please submit a pdf of your worksheet(s).] *[2]*

SOLUTION. Please see the pdf of the `Maple` worksheet appended to the solutions for all the plots. *[Oops! Forgot the third parametric curve ... 0.5 points off – would have been 1 point off except that I did giave the informal description.]*

Again, note that the plots do not necessarily have x and y plotted to the same scale, which will distort the curves in many cases. That said, the first curve is the unit circle centred at the origin; the second is a spiral starting at the origin and ending at 2π on the x-axis; and the third is a semi-ellipse, being the lower half of the ellipse with (horizontal) semi-major axis 3 and (vertical) semi-minor axis 1, centred at the origin. ■

Polar coordinates are an alternative to the usual two-dimensional Cartesian coordinates. The idea is to locate a point by its distance r from the origin and its direction, which is given by the (counterclockwise) angle θ between the positive x-axis and the line from the origin to the point. Thus, if (r, θ) are the polar coordinates of some point, then its Cartesian coordinates are given by $x = r\cos(\theta)$ and $y = r\sin(\theta)$. (Note that for purposes of calculus it is usually more convenient to measure angles in radians rather than degrees.) Polar coordinates come in particularly handy when dealing with curves that wind around the origin, since such curves can often be conveniently represented by an equation of the form $r = f(\theta)$ for some function f of θ. If r is negative for a given θ, we interpret that as a distance of $|r|$ in the *opposite* direction, *i.e.* the direction $\theta + \pi$.

4. Use `Maple` to separately plot the curves in polar coordinates given by $r = 1$ for $0 \leq \theta \leq 2\pi$, by $r = \theta$ for $0 \leq r \leq 2\pi$, and by $r = 2 - 2\cos(\theta)$ for $0 \leq \theta \leq 2\pi$. Describe each of these curves informally. [Please submit a printout of your worksheet(s).] *[2]*

SOLUTION. Please see the pdf of the `Maple` worksheet appended to the solutions for all the plots. Note that the `plots` package, which is required to use the `polarplot` command was loaded already in order to use the `implicitplot` command for **2**.

Note that none of the polar plots given have different scales for x and y, so they are not distorted. (Given the nature of polar plots, you are unlikely to need this to make the plot fit into a standard-sized box, which `Maple` tries to do by default.) The first curve is an unit circle centred at the origin; the second is a spiral starting at the origin and ending at 2π on the x-axis; and the third is another cardioid with x-intercepts at $x = -4$ and $x = 0$ and y-intercepts at $y = -2$, $y = 0$, and $y = 2$. ■

5. Some of the curves in problems **1**–**4** are all of or parts of other curves in problems **1**–**4**, obviously with different presentations. Identify all of the curves related in this way that you can. *[2]*

SOLUTION. Here they are:

(a) The curves

2. $x^2 + y^2 = 1$,
3. $x = \cos(t)$ and $y = \sin(t)$ for $0 \le t \le 2\pi$, and
4. $r = 1$ for $0 \le \theta \le 2\pi$

are all the circle of radius 1 (*i.e.* unit circle) centred at the origin. The curve

1. $y = \sqrt{1 - x^2}$ for $-1 \le x \le 1$ is the upper half of the same circle.

(b) The curves

1. $y = -\frac{1}{3}\sqrt{9 - x^2}$ for $-3 \le x \le 3$ and
3. $x = 3\cos(t)$ and $y = \sin(t)$ for $\pi \le t \le 2\pi$

are both the lower half of the ellipse with (horizontal) semi-major axis 3 and (vertical) semi-minor axis 1, centred at the origin. The curve

2. $x^2 + 9y^2 = 9$

is the entire ellipse with (horizontal) semi-major axis 3 and (vertical) semi-minor axis 1, centred at the origin.

(c) The curves

2. $\left(x^2 + y^2\right)^2 + 4x\left(x^2 + y^2\right) - 4y^2 = 0$ and
4. $r = 2 - 2\cos(\theta)$ for $0 \le \theta \le 2\pi$

are the same cardioid, which has x-intercepts at $x = -4$ and $x = 0$ and y-intercepts at $y = -2$, $y = 0$, and $y = 2$.

(d) The curves

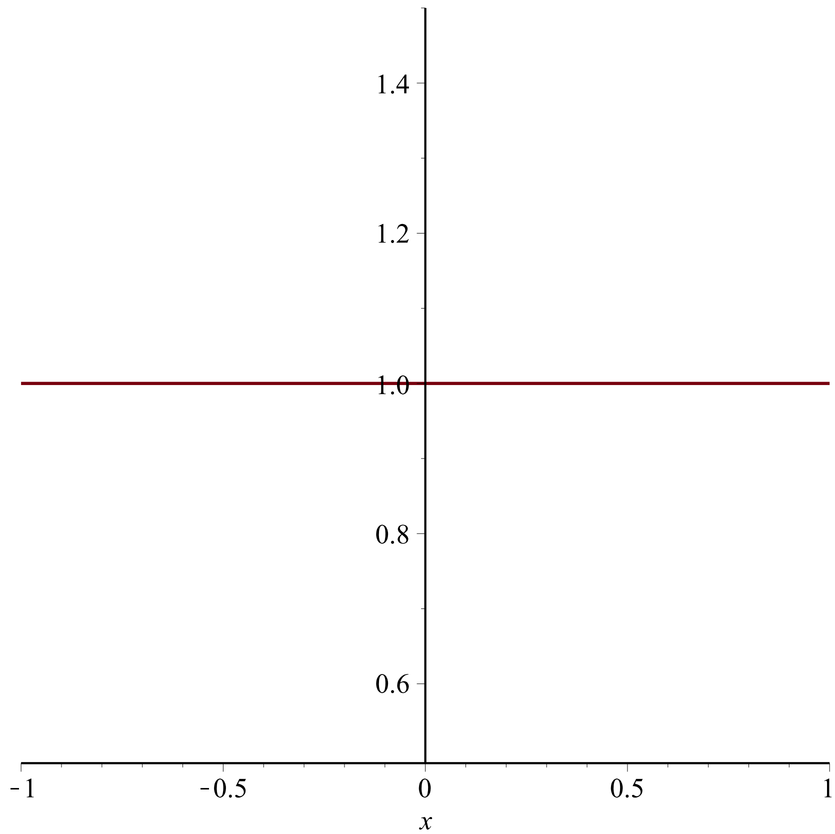
3. $x = t\cos(t)$ and $y = t\sin(t)$ for $0 \le t \le 2\pi$ and
4. $r = \theta$ for $0 \le r \le 2\pi$

are the same spiral starting at the origin and ending at 2π on the x-axis. ■

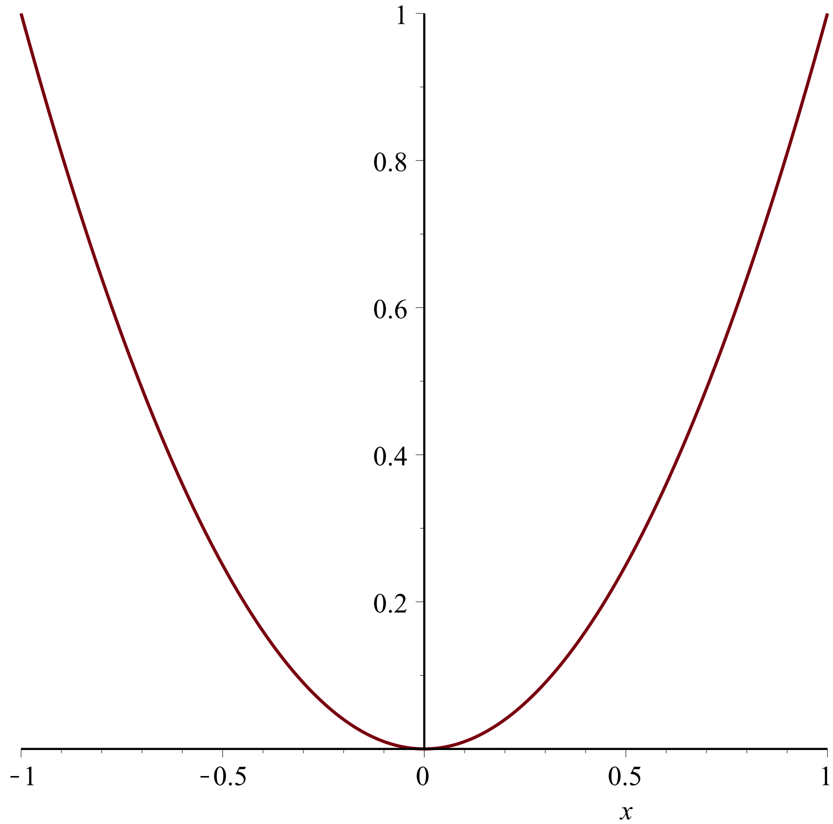
REFERENCES

1. *A very quick start with* `Maple`, by Stefan Bilaniuk, which can be found (pdf) on Blackboard or the course archive page at: at: `euclid.trentu.ca/math/sb/1110H`
2. *Formula for Success*, by Ruth Brandow, Ellen Dempsey, Marj Tellis, and Lisa Davies, Academic Skills Centre, Trent University, which can be found (pdf) at: `www.trentu.ca/academicskills/documents/ASC_mathematics_000.pdf`
3. *Single Variable Calculus* (Early Transcendentals), by David Guichard, licensed under the Creative Commons Attribution-NonCommercial-ShareAlike License. Available on Blackboard and the course archive page at: `euclid.trentu.ca/math/sb/1110H` May also be downloaded for free from: `communitycalculus.org`
4. *Getting started with Maple 10*, by Gilberto E. Urroz (2005), which can be found (pdf) on Blackboard or the course archive page at: `euclid.trentu.ca/math/sb/1110H` *Thanks to Prof. Urroz for permission to use it!*

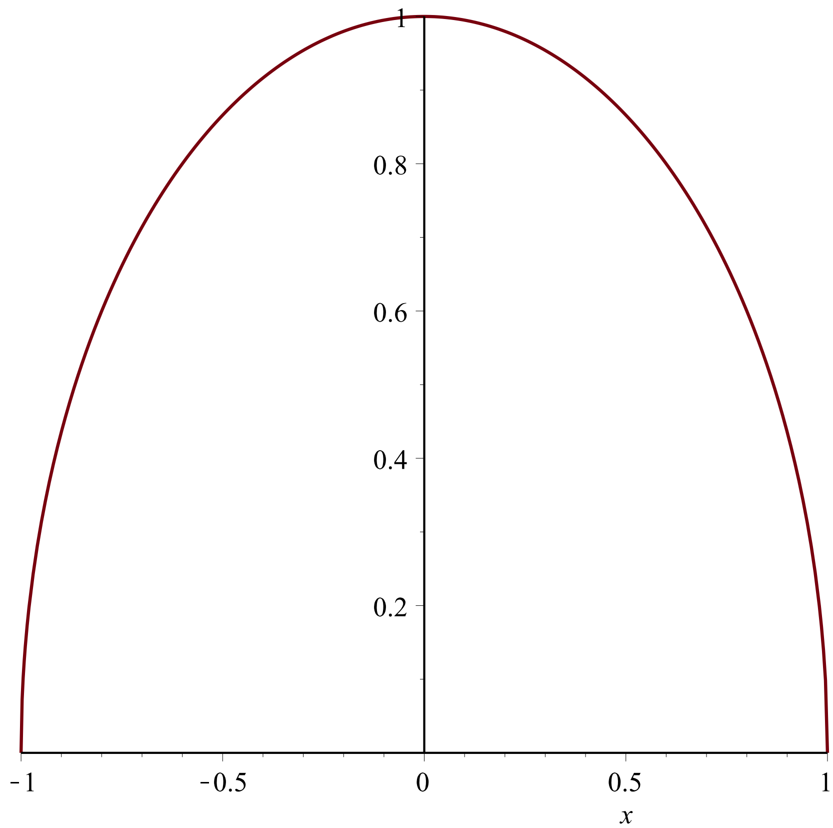
> $plot(1, x = -1 ..1)$



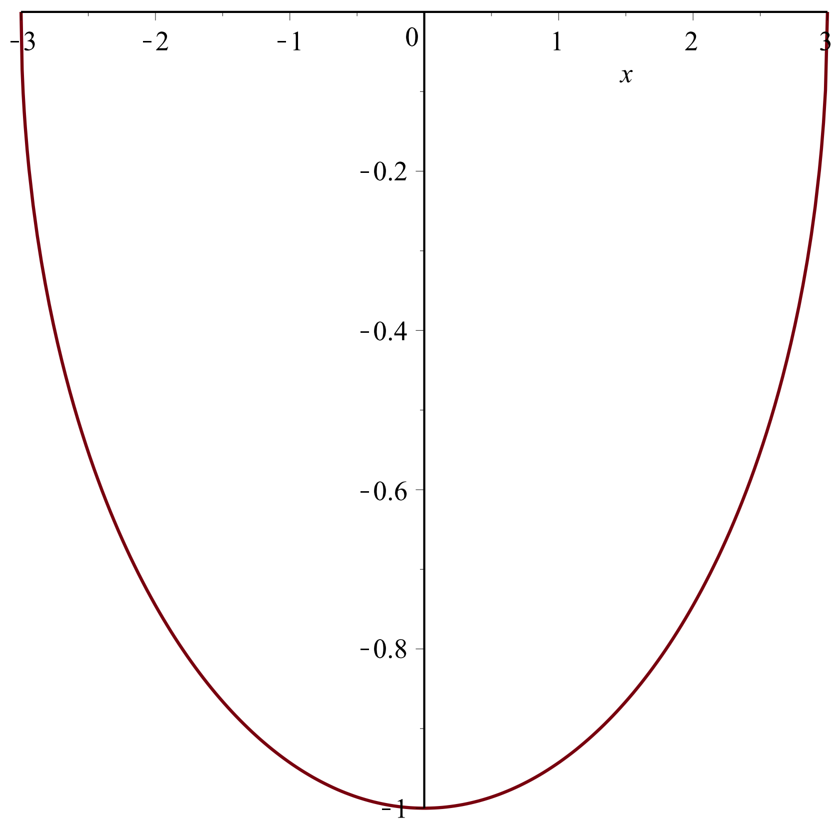
> $plot(x^2, x = -1 ..1)$



> $plot(\mathrm{sqrt}(1 - x^2), x = -1 ..1)$



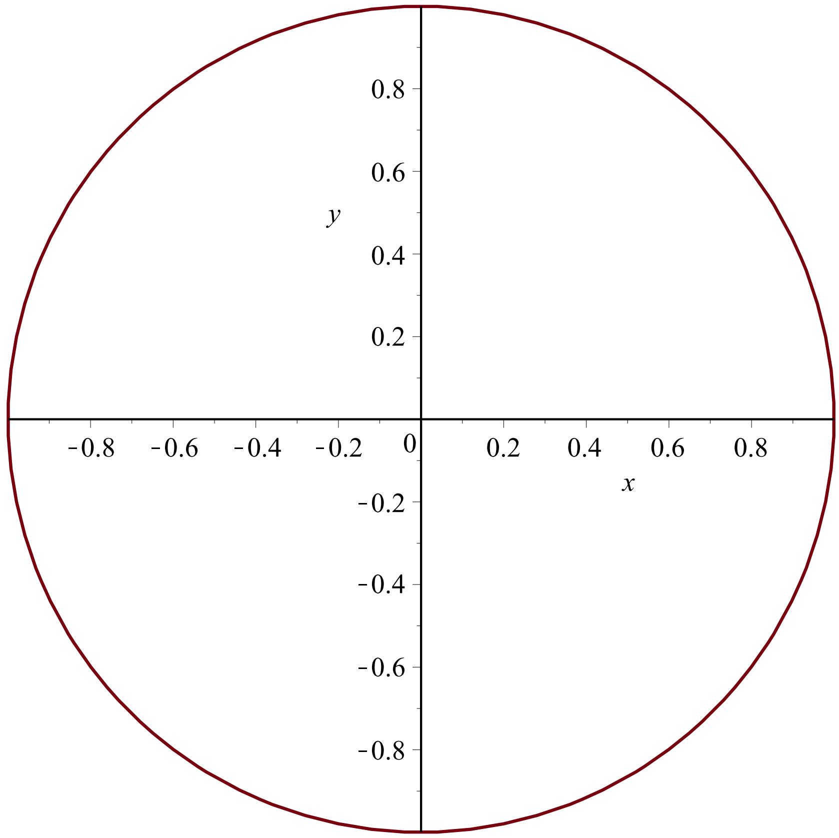
> $plot\left(-\left(\frac{1}{3}\right)\cdot \mathrm{sqrt}\left(9-x^2\right), x=-3..3\right)$



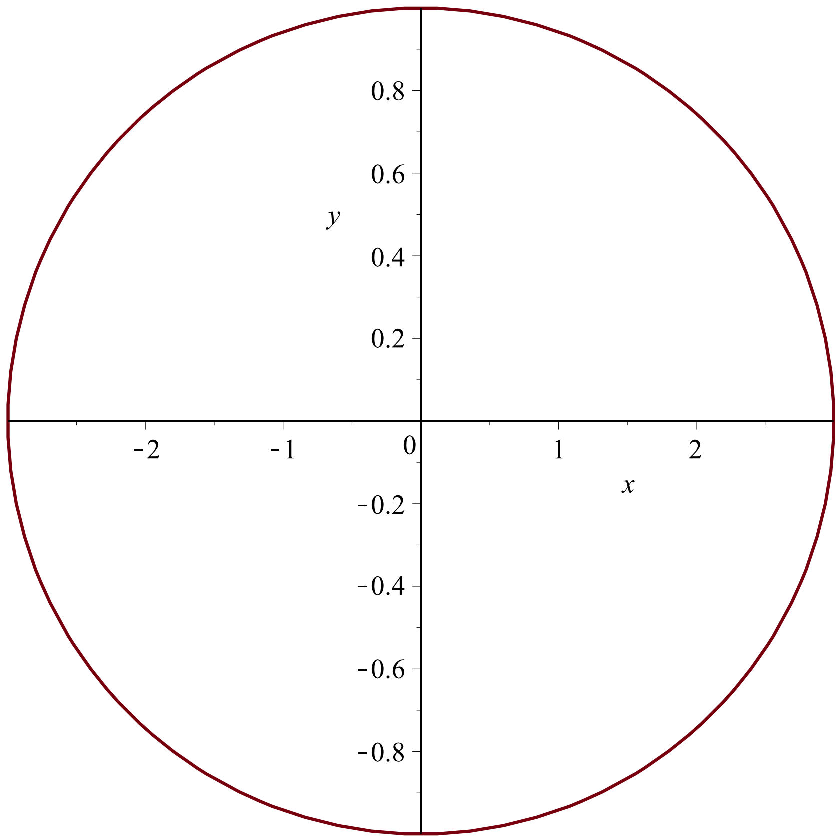
> *with*(*plots*)

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(1)**

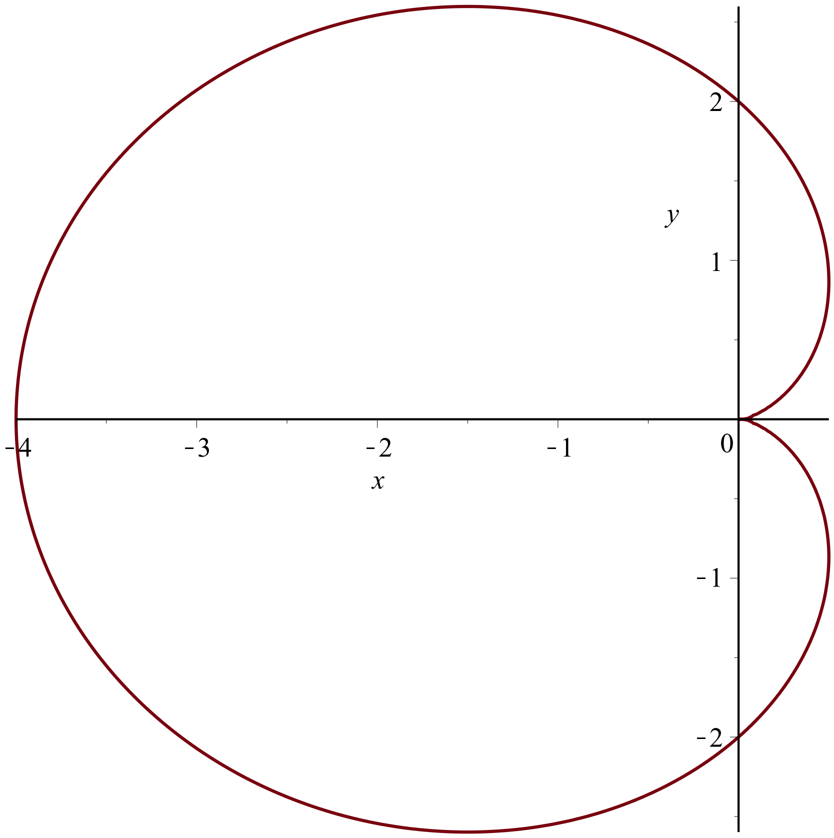
> *implicitplot*$\left(x^2 + y^2 = 1, x = -1 ..1, y = -1 ..1\right)$



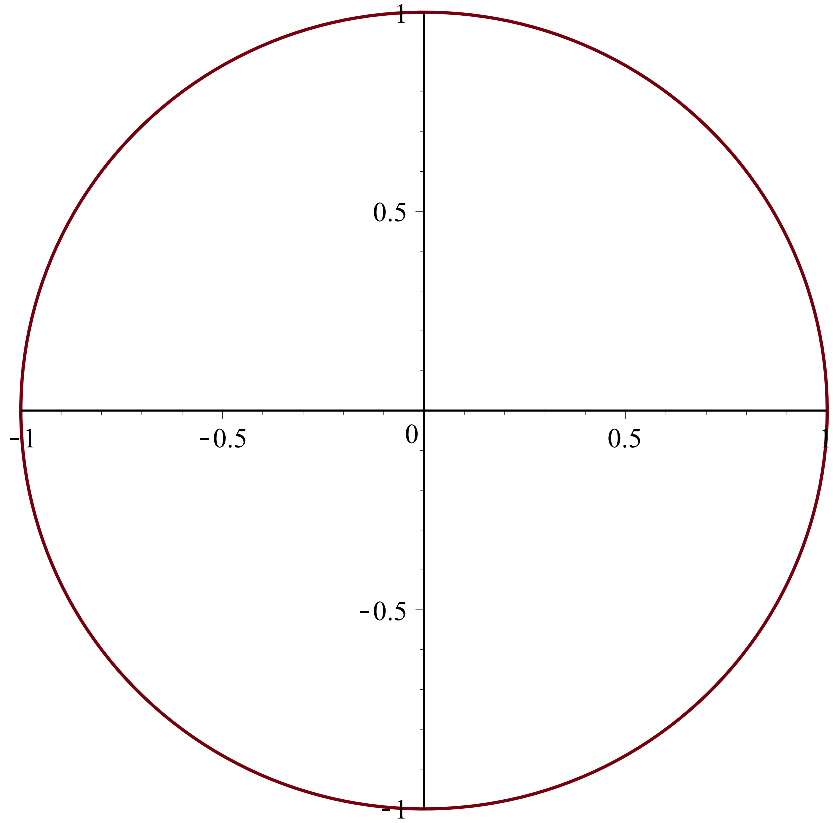
> $implicitplot(x^2 + 9 \cdot y^2 = 9, x = -3 ..3, y = -1 ..1)$



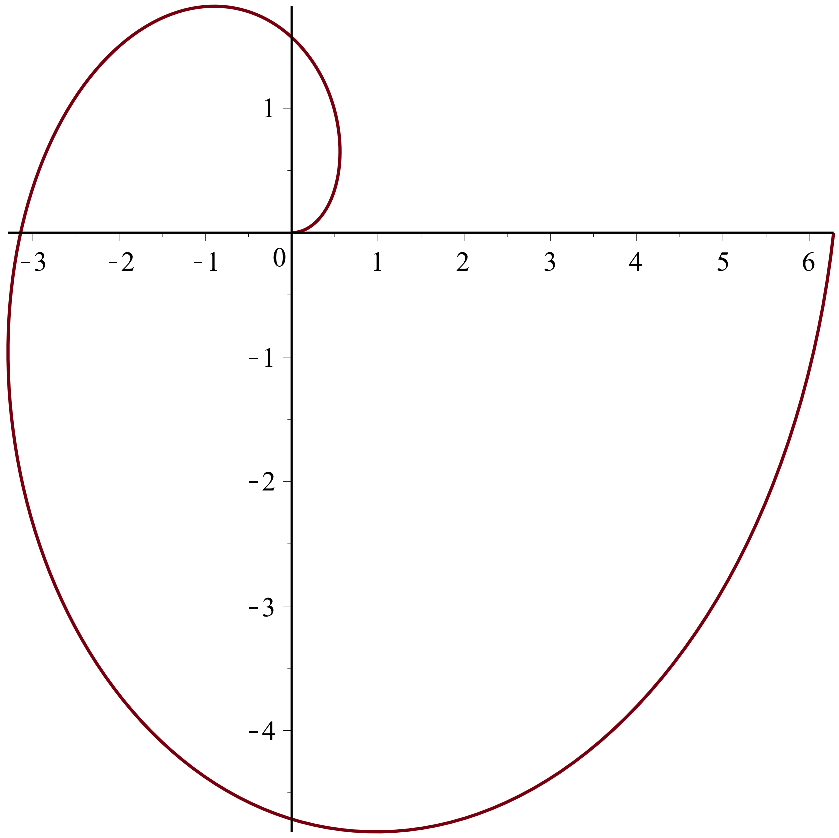
> $implicitplot\left(\left(x^2+y^2\right)^2+4\cdot x\cdot\left(x^2+y^2\right)-4\cdot y^2=0, x=-5..5, y=-5..5, gridrefine=4\right)$



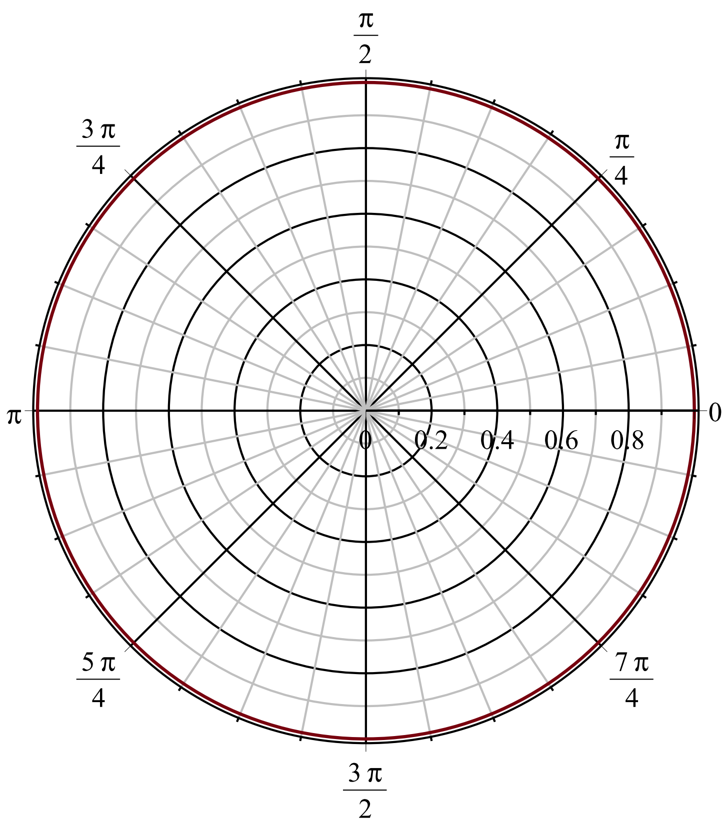
> $plot([\cos(t), \sin(t), t = 0..2 \cdot \mathrm{Pi}])$



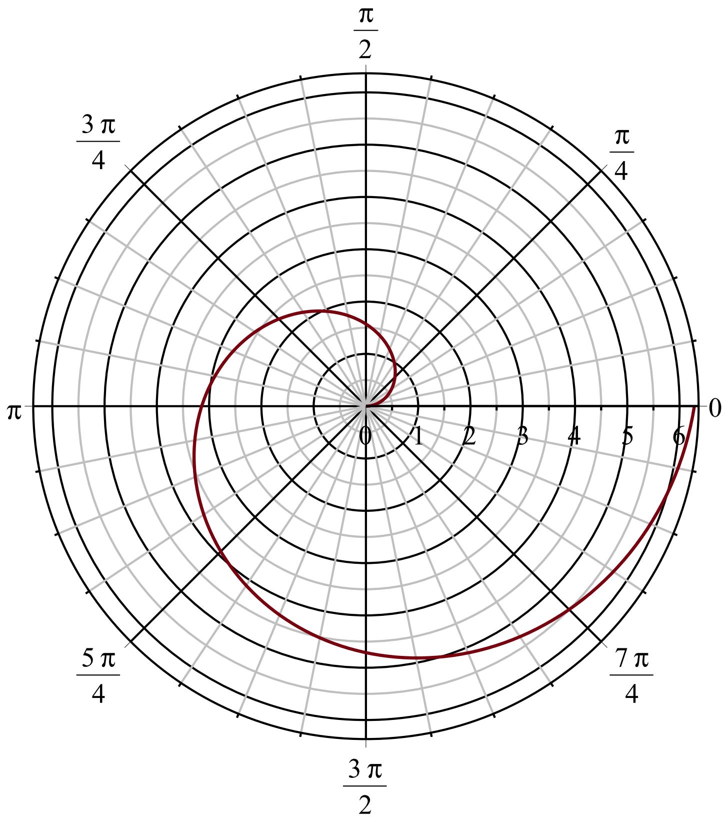
> $plot([t \cdot \cos(t), t \cdot \sin(t), t = 0 .. 2 \cdot \mathrm{Pi}])$



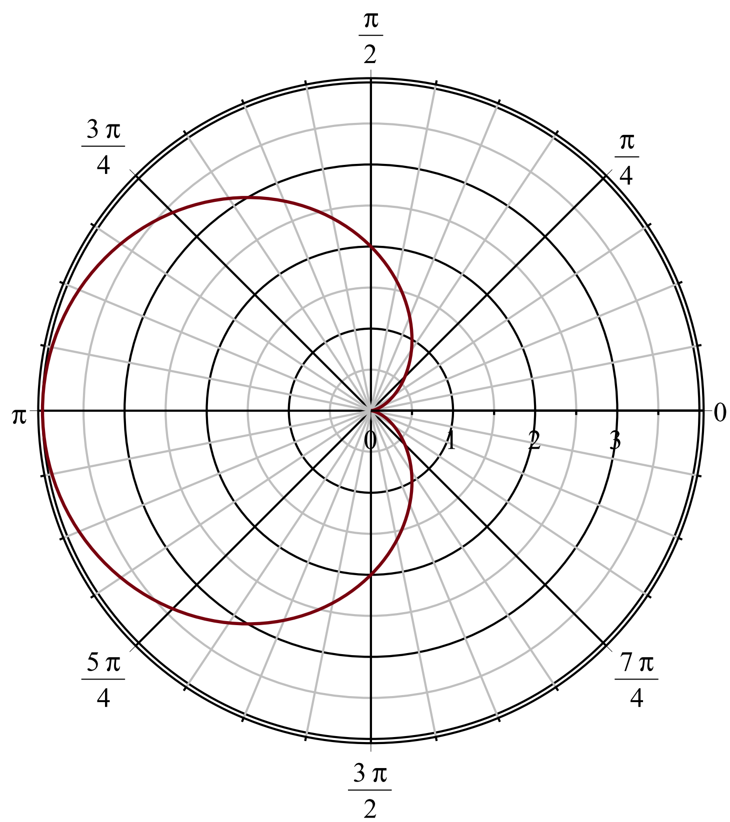
> *polarplot*(1, theta = 0 ..2 · Pi)



> *polarplot*(theta, theta = 0 ..2 · Pi)



> *polarplot*(2 − 2·cos(theta), theta = 0 ..2·Pi)



π/2
3π/4
π/4
π
0
0
1
2
3
5π/4
7π/4
3π/2

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