**Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals**
Trent University, Fall 2020
**Solution to Quiz #9 - The Escape of the Ant**
*Tuesday, 24 November .*

A box with dimensions 12 *cm* by 12 *cm* by 30 *cm* is sitting on a desk, with the bottom being one of the 12 *cm* by 30 *cm* faces. The box has a small hole in one of the 12 *cm* by 12 *cm* faces, with the hole being in the middle 1 *cm* below the top of that face. An ant is inside the box, initially in the middle of the other 12 *cm* by 12 *cm* face, 1 *cm* above the bottom of that face. The ant wants to leave the box by the shortest path it can, but it can only move on the faces of the box. (No jumping, no flying, no teleportation . . . :-)
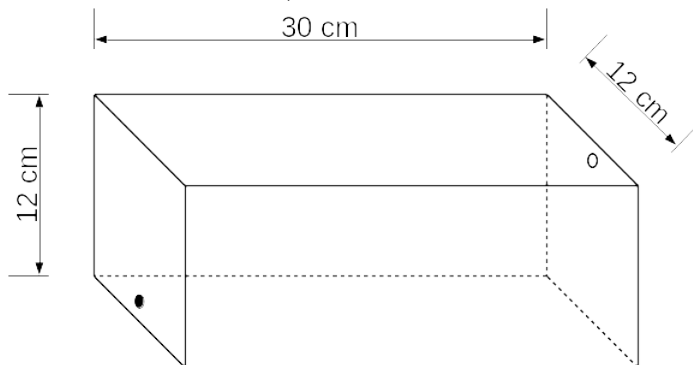


**1.** What is the length of the shortest path along the sides and/or top and/or bottom of the given box from the initial position of the ant to the hole? *[5]*

SOLUTION. The shortest path from point to another along a surface is a straight line if that surface is flat. Unfold the box to make it a flat surface as in the diagram below:



1
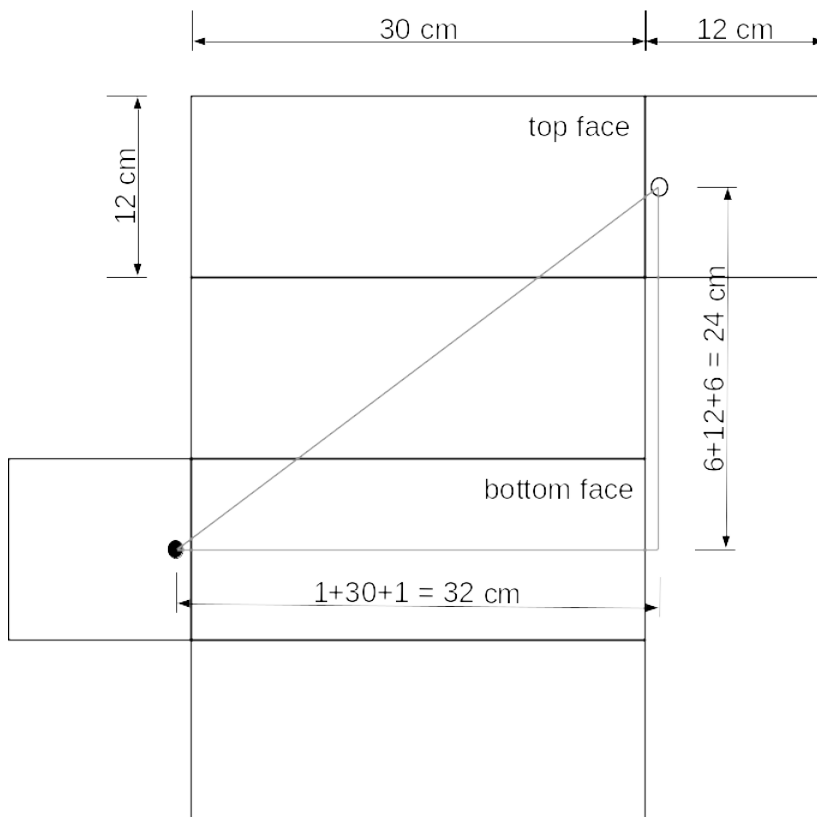
Since the ant is constrained to move along the faces of the box, each of which is flat even before the box was unfolded, the shortest path from its initial position to the hole has the same length whether the box is unfolded or not. In the unfolded state, however, the path, which is just a straight line, is much easier to spot.

As noted in the diagram, the path is the hypotenuse of a right triangle with base $32$ $cm$ and height $24$ $cm$, so it has length

$$\sqrt{32^2 + 24^2} = \sqrt{1024 + 576} = \sqrt{1600} = 40 \ cm \,. \qquad \blacksquare$$

NOTE: The original version of this problem is due to Henry Ernest Dudeney (1857-1930), an English author and mathematician who devised many mathematical puzzles and games. I found the problem in a chapter on Dudeney and his puzzles in the *Second Book of Mathematical Puzzles & Diversions* (1961), which is the second collection of Martin Gardner's (1914-2010) *Mathematical Games* columns from *Scientific American*. Gardner wrote that column each month from 1957 through 1980, and contributed to it intermittently thereafter until 1986.