Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
Trent University, Fall 2018
Solutions to Assignment #7
Time to Hike
or,
A Change of Pace

Meredith Mathie goes on a hike in the mountains, setting out from the Origin Lodge at 9 a.m. and taking a trail that winds over hill and dale through `Maple` Forest to Maximum Peak. On reaching the summit of Maximum Peak, Meredith immediately turns around and, following exactly the same trail in reverse, returns to Origin Lodge at 5 p.m. Meredith is a tad obsessive-compulsive and always walks at a speed of 4.5 km/h going uphill, 6 km/h on level ground, and 9 km/h going downhill. At no point between departing from and returning to Origin Lodge does Meredith take a break or pause.

1. What is the round-trip distance walked by Meredith? *[5]*

SOLUTION. The key to this problem, besides the information about the speeds on different slopes, is the fact that Meredith retraces *exactly* same path when returning to Origin Lodge that was taken when hiking out to Maximum Peak. How is this significant? Consider, hypothetically, a 1 km stretch of the path consisting entirely of level ground. It takes $\frac{1}{6}$ hr to cover this stretch heading out to Maximum Peak, and another $\frac{1}{6}$ hr to cover it again when returning, for a total of $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$ hr to cover it on the round trip. By contrast, a 1 km stretch of the path that is uphill [respectively, downhill] when heading out is downhill [respectively, uphill] when returning. It takes $\frac{1}{4.5} = \frac{2}{9}$ hr to cover this stretch when going uphill and $\frac{1}{9}$ hr when going downhill, for a total of $\frac{2}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$ hr to cover this stretch on the round trip.

It follows from the arithmetic above that each part of the path takes the same amount of time to traverse on the round trip as any other stretch of the same length, irrespective of the mix of level ground, uphill, and downhill that stretch consists of. If each kilometre of path requires $\frac{1}{3}$ hr to cover on the round trip, then each hour of the round trip covers 3 km going out and coming back, for a total of 6 km/hr. Since the round-trip hike takes 8 hr, the round trip distance must be $6 \cdot 8 = 48$ km. ■

2. Determine, as accurately as you can, at what time Meredith reached the summit of Maximum Peak. *[5]*

SOLUTION. From the solution to problem **1** we know that the summit of maximum Peak is half the round trip distance from Origin Lodge, that is, $\frac{48}{2} = 24$ km.

Since going downhill is fastest, the earliest Meredith could reach the summit would be if the path from Origin Lodge to the summit of Maximum Peak was all downhill. (Yes, Maximum Peak would be ever so slightly misnamed if this was the case . . . :-) In this case, hiking from Origin Lodge to the summit of Maximum Peak would require covering each of the 24 km in $\frac{1}{9}$ hr/km, for a total of $\frac{24}{9} = \frac{8}{3}$ $hr = 2$ hr 40 min. This would mean that the earliest that Meredith could have reached the summit of Maximum Peak is at 11:40 a.m.

Similarly, since going uphill is slowest, the latest Meredith could reach the summit would be if the path from Origin Lodge to the summit of Maximum Peak was all uphill.

In this case, hiking from Origin Lodge to the summit of Maximum Peak would require covering each of the 24 km in $\frac{2}{9}$ hr/km, for a total of $24 \cdot \frac{2}{9} = \frac{16}{3}$ $hr = 5$ hr 20 min. This would mean that the earliest that Meredith could have reached the summit of Maximum Peak is at 2:20 p.m.

Putting these together, we see that Meredith must have reached the summit of Maximum Peak sometime between 11:40 a.m. and 2:20 p.m. inclusive. ∎

NOTE. This problem was ~~stolen~~ adapted from *Knot I* of Lewis Carroll's *A Tangled Tale*, which was originally serialized in a magazine, starting in April of 1880. (Yes, this is the same Lewis Carroll – real name Charles Lutwidge Dodgson – who wrote *Alice in Wonderland*. You can find it online, for example at `www.gutenberg.org/ebooks/29042` .