Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
Trent University, Fall 2018

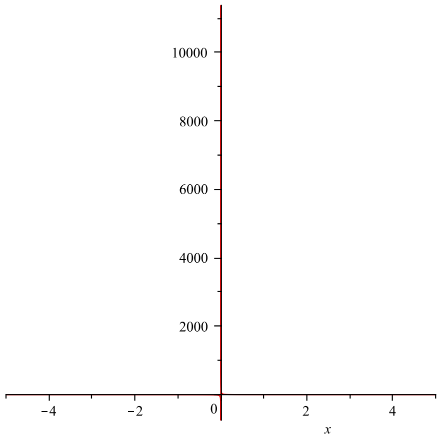
Solutions to Assignment #1
Plotting with Maple[†]

1. Use `Maple` to plot the graphs (separately!) of each of the following functions: $y = \frac{1}{x}$, $y = \sqrt{x}$, $y = \sin(x)$, and $y = \cos(x)$, all for $-5 \leq x \leq 5$. *[2]*

Solution. Suitable instances of the `plot` command are given below. The output graphs have been shrunk to save page space, but are otherwise unaltered.

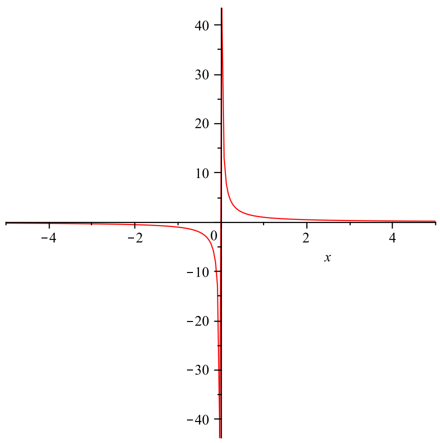
It was very mean of your instructor to put the function $y = \frac{1}{x}$ as the very first one to plot, because typing in what simply ought to work gives something a little ugly:

```
> plot(1/x, x = -5..5)
```



This problem can be corrected, ironically enough, by turning off a feature that `Maple` uses by default to make graphs look good:

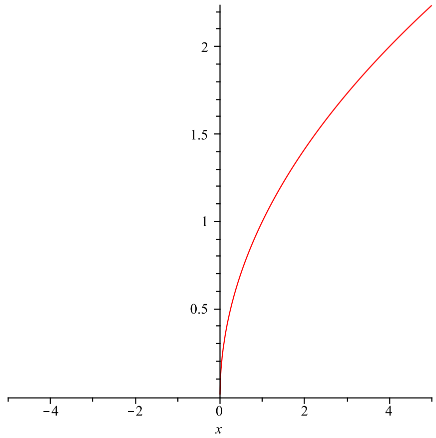
```
> plot(1/x, x = -5..5, adaptive=false)
```



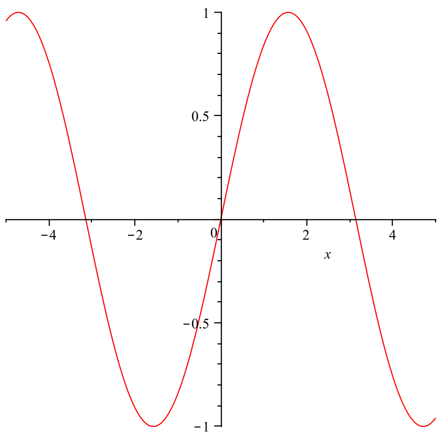
[†] . . . even as `Maple` plots against you!

Fortunately, the other three functions do look well enought by default.

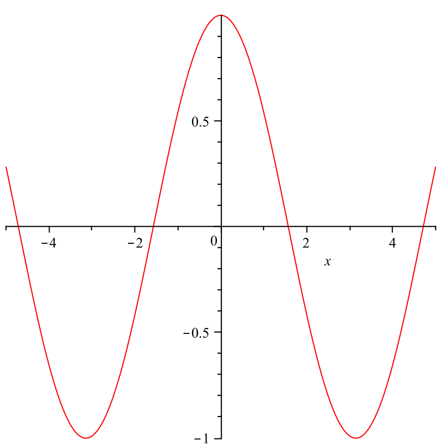
```
> plot(sqrt(x), x = -5..5)
```




```
> plot(sin(x), x = -5..5)
```



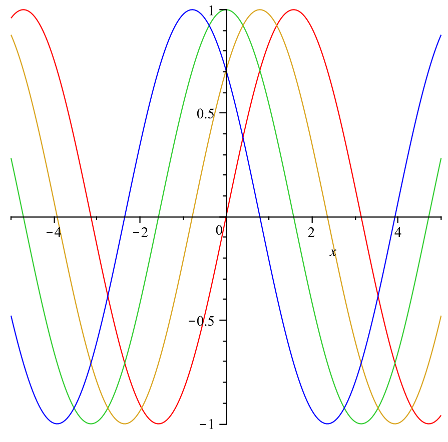

```
> plot(cos(x), x = -5..5)
```



2. Use `Maple` to plot the graphs of each of the following functions, all together in one picture: $y = \sin(x)$, $y = \cos(x)$, $y = \sin\left(x + \frac{\pi}{4}\right)$, and $y = \cos\left(x + \frac{\pi}{4}\right)$, all for $-5 \leq x \leq 5$. *[2]*

SOLUTION. This one looks pretty:

$>$ `plot([sin(x), cos(x), sin(x+Pi/4), cos(x+Pi/4)], x = -5..5)`

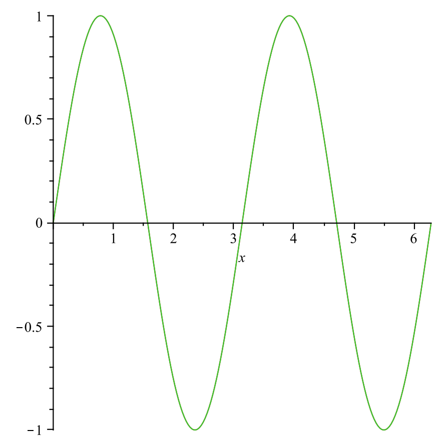


■

3. Use `Maple` to plot the graphs of $y = \sin(2x)$ and $y = 2\sin(x)\cos(x)$ over a suitable range of xs. Explain why you think this range is suitable. Does your output support the formula $\sin(2x) = 2\sin(x)\cos(x)$ or not? *[2]*

SOLUTION. Since $\sin(x)$ and $\cos(x)$ are periodic with period 2π and $\sin(2x)$ is periodic with period π [Why?], a range of 0 to 2π will show all that either function to be plotted does. If you plot both functions at the same time,

$>$ `plot([sin(2*x), 2*sin(x)*cos(x)], x = 0..2*Pi)`

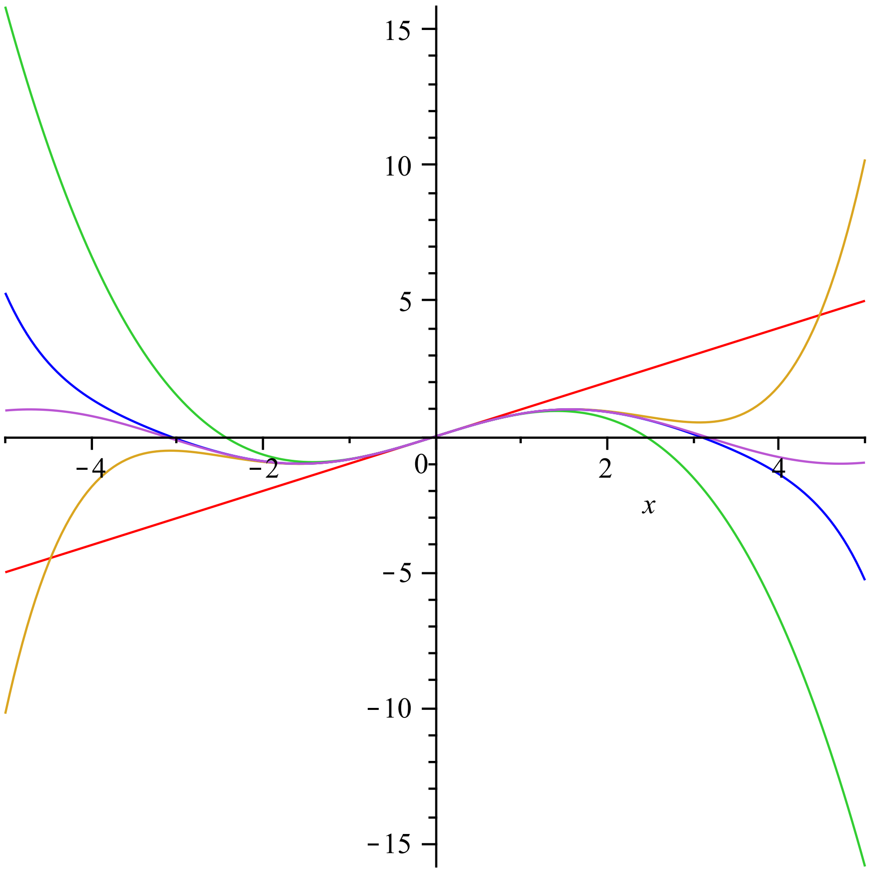


… they overlap perfectly, which supports the hypothesis that the formula $\sin(2x) = 2\sin(x)\cos(x)$ is correct. (Which it is, of course.) ■

4. Use `Maple` to plot the graphs of each of the following functions, all together in one picture: $y = x$, $y = x - \frac{x^3}{3!}$, $y = x - \frac{x^3}{3!} + \frac{x^5}{5!}$, $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$, and $y = \sin(x)$, all for $-5 \leq x \leq 5$. What pattern(s) can you discern from looking at these graphs? How could these patterns be used? *[4]*

SOLUTION. Here is the plot, with less shrinking to make it easier to pick out details:

```
> plot([x, x-x^3/3!, x-x^3/3!+x^5/5!, x-x^3/3!, x-x^3/3!+x^5/5!-x^7/7!,
  sin(x)] , x = -5..5)
```



The pattern isn't too hard to spot: $y = x$ is close to $y = \sin(x)$ near 0, $y = x - \frac{x^3}{3!}$ is close to $y = \sin(x)$ for a larger interval around 0, $y = x - \frac{x^3}{3!} + \frac{x^5}{5!}$ is close to $y = \sin(x)$ for an even larger interval around 0, and $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ gets close to $y = \sin(x)$ for a yet larger interval around 0. This trick can be used to approximate $\sin(x)$ to whatever precision you need by evaluating a suitable polynomial of high enough degree. More on that in MATH 1120H when we do Taylor polynomials and Taylor series! ■