Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2018

Assignment #4
It's a cinch!?
Due on Friday, 12 October.

Recall from class or the textbook that the basic hyperbolic functions are

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad \text{and} \quad \cosh(x) = \frac{e^x + e^{-x}}{2}\,.$$

We can define the other hyperbolic functions from these in the same way that we define the other trigonometric functions from $\sin(x)$ and $\cos(x)$. In particular,

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad \text{and} \quad \operatorname{sech}(x) = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}\,.$$

Like the trigonometric functions, the hyperbolic functions can be inverted, albeit sometimes only partially. The main task in this assignment is to invert $\tanh(x)$.

1. Plot $y = \tanh(x)$. *[1]*
2. What are the domain and range of $\tanh(x)$? *[1]*
3. Find a formula for $\operatorname{arctanh}(x)$, the inverse function of $\tanh(x)$, by hand. What are the domain and range of $\operatorname{arctanh}(x)$? *[4]*
4. Use `Maple` to find a formula for $\operatorname{arctanh}(x)$. *[1]*
5. Find the derivative of $\operatorname{arctanh}(x)$ by hand, and then by using `Maple`. How does it compare to the derivative of $\arctan(x)$? *[3]*