

Mathematics 1101Y – Calculus I: functions and calculus of one variable

TRENT UNIVERSITY, 2013–2014

Test #1

Monday, 11 November, 2013.

Time: 50 minutes

Instructions

- Show all your work. Legibly, please!
- If you have a question, ask it!
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator and an aid sheet.

1. Find $\frac{dy}{dx}$ as best you can in any *three* (3) of **a–e**. [12 = 3 × 4 each]

a. $y = x \tan(x)$ b. $y = \frac{e^x}{x}$ c. $1 = \ln(xy)$ d. $y = \sin^3(x + 41)$ e. $y = \frac{1}{1 + \sqrt{x}}$

2. Do any *two* (2) of **a–d**. [10 = 2 × 5 each]

a. Find the intercepts and the coordinates of the vertex of the parabola $y = x^2 - 2x - 3$.

b. Compute $\lim_{x \rightarrow 0} \frac{x^2}{\sin(x)}$.

c. Find $f^{-1}(x)$ for $f(x) = \frac{1}{1 + \sqrt{x}}$.

d. Use the limit definition of the derivative to find $f'(1)$ if $f(x) = x^2 + x$.

3. Do *one* (1) of **a** or **b**. [8]

a. A Borg cube's volume expands proportionately to how much matter it ingests: every 100 kg of matter ingested adds 1 m³ to the volume. If the Borg cube ingests matter at a constant rate of 3000 kg/s, how quickly is each side of the cube growing at the instant that each side of the cube measures 10 m?

b. What is the maximum area of a rectangle whose total perimeter is 16 m?

4. Find the domain and all the intercepts, vertical and horizontal asymptotes, maxima and minima, and points of inflection of $f(x) = \frac{x^2 + 1}{x}$, and sketch its graph. [10]

[Total = 40]