Mathematics 1101Y – Calculus I: functions and calculus of one variable
TRENT UNIVERSITY, 2012–2013
Final Examination

Time: *09:00–12:00, on Thursday, 11 April, 2013.* *Brought to you by* Стефан Біланюк.
Instructions: Do parts **I**, **J**, and **K**, and, if you wish, part **Z**. Show all your work and justify all your answers. *If in doubt about something,* **ask!**
Aids: Any calculator; (all sides of) one aid sheet; one (1) brain ($10^{10^{10}}$ neuron limit).

Part I. Do all four (4) of **1–4**.

1. Compute $\dfrac{dy}{dx}$ as best you can in any *three* (3) of **a–f**. *[15 = 3 × 5 each]*

a. $y = \dfrac{e^{2x}-1}{e^{2x}+1}$ **b.** $y = \arctan(t)$, $x = \dfrac{1}{3}t^3 + t$ **c.** $y = (1+\sin(x))^2$

d. $\tan(y) = x$ **e.** $y = xe^{-x}$ **f.** $y = \displaystyle\int_1^x \frac{\ln(t)}{t}\,dt$

2. Evaluate any *three* (3) of the integrals **a–f**. *[15 = 3 × 5 each]*

a. $\displaystyle\int \sec^{17}(x)\tan(x)\,dx$ **b.** $\displaystyle\int_0^{\sqrt{\pi}} z\cos\left(z^2\right)\,dz$ **c.** $\displaystyle\int \frac{1}{\sqrt{4+x^2}}\,dx$

d. $\displaystyle\int_0^1 \arctan(y)\,dy$ **e.** $\displaystyle\int \frac{1}{x^3+x}\,dx$ **f.** $\displaystyle\int_1^{\infty} \frac{1}{t^2}\,dt$

3. Do any *three* (3) of **a–f**. *[15 = 3 × 5 each]*

a. Find the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n^2}x^n$.

b. Sketch the polar curve $r = \theta$, $0 \le \theta \le \pi$, and find the area of the region between this curve and the origin.

c. Determine whether the series $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n}}{(n+1)^2}$ converges or diverges.

d. Sketch the region between $y = x^2$ and $y = \sqrt{x}$, $0 \le x \le 1$, and find its area.

e. Sketch the parametric curve $x = \cos(t)$, $y = \sin(t)$, $0 \le x \le \pi$, and find its arc-length.

f. Compute $f'(0)$ using the limit definition of the derivative if $f(x) = x^2 + x + 1$.

g. Sketch the solid obtained by revolving the region between $y = 1$ and $y = \sqrt{x}$, $0 \le x \le 1$, about the y-axis, and find its volume.

4. Consider the curve $y = \dfrac{x^2}{2}$, for $0 \le x \le 2$.

a. Sketch the curve. *[1]*

b. Sketch the surface obtained by revolving the curve about the x-axis. *[1]*

c. Compute either *i.* the length of the curve or *ii.* the area of the surface. [Just one, please!] *[8]*

Part J. Do any *two* (2) of **5–7**. *[30 = 2 × 15 each]*

5. Gravel is dumped from a conveyor belt at a rate of 3 m^3/min. At any given instant the gravel forms a conical pile whose height is twice the radius of the base. How fast is the height of the pile increasing at the instant that the pile is 1 m high? [The volume of a cone with height h and base radius r is $\frac{1}{3}\pi r^2 h$.]

6. Find any and all intercepts, maximum, minimum, and inflection points, and vertical and horizontal asymptotes of $f(x) = e^{1/x}$, and sketch its graph.

7. Sketch the solid obtained by revolving the region between $y = x$ and $y = x^2$, for $0 \leq x \leq 1$, about the line $x = -2$ and find its volume.

Part K. Do *one* (1) of **8** or **9**. *[15 = 1 × 15 each]*

8. Let $f(x) = \dfrac{1}{(2+x)^2}$.

- **a.** Use Taylor's formula to find the Taylor series at 0 of $f(x)$. *[10]*
- **b.** Find the radius and interval of convergence of this Taylor series. *[5]*
- **c.** *[Bonus!]* Find the Taylor series at 0 of $f(x)$ without using Taylor's formula. *[1]*

9. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (z-2)^n}{2^n}$, where z is an unknown.

- **a.** Determine for which values of z the series converges. *[10]*
- **b.** Find a function $g(z)$ equal to this series when it converge. *[5]*

[Total = 100]

Part Z. Bonus problems! Do them (or not – less for me to mark! :-), if you feel like it.

0. Recall that an integer greater than 1 is a prime number if it has no positive integer factors other than itself and 1. Does the polynomial $p(x) = x^2 + x + 41$ always give you a prime number as its output whenever x is an integer greater than or equal to zero? Explain why or why not. *[1]*

00. Write a haiku touching on calculus or mathematics in general. *[2]*

haiku?

seventeen in three:
five and seven and five of
syllables in lines

I HOPE YOU HAVE EVEN MORE FUN THIS SUMMER
THAN YOU DID IN THIS COURSE!