Mathematics 1101Y – Calculus I: Functions and calculus of one variable
TRENT UNIVERSITY, 2011–2012

Solutions to Assignment #4
Definite integrals with `Maple`

For this assignment, look up `Maple`'s basic integration command, `int`, as well as the commands for evaluating an expression as a decimal, `evalf`, and for solving an equation numerically, `fsolve` (which works pretty much like `solve` otherwise).

The latter part of this assignment is concerned with the function $f(x) = e^{-x^2}$, which does not have a nice antiderivative. However, we will start with things you can do by hand for a warmup.

1. Compute $\displaystyle\int_0^1 x^2\,dx$ both by hand and using `Maple`. *[2]*

SOLUTION. By hand:

$$\int_0^1 x^2\,dx = \left.\frac{x^3}{3}\right|_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3} - 0 = \frac{1}{3}$$

Using `Maple`:

```
> int(x^2,x=0..1);
```

$$\frac{1}{3}$$

Not much to it either way ... ■

2. Find the value of t such that $\displaystyle\int_0^t x^2\,dx = 9$ both by hand and using `Maple`. *[2]*

SOLUTION. By hand:

$$\int_0^t x^2\,dx = \left.\frac{x^3}{3}\right|_0^t = \frac{t^3}{3} - \frac{0^3}{3} = \frac{t^3}{3}\,,$$

so the problem boils down to solving the equation $\dfrac{t^3}{3} = 9$. Then $t^3 = 3 \cdot 9 = 27 = 3^3$, so $t = 3$.

Using `Maple`:

```
> fsolve(int(x^2,x=0..t)=9,t);
```

$$3.$$

Perhaps just a bit easier with `Maple` this time. ■

3. Use `Maple` to find $\int_0^{\pi} x^2\,dx$ to 10 decimal places. *[1]*

SOLUTION.

```
> evalf(int(x^2,x=0..Pi));
```

$$10.33542556$$

I'd rather not compute $\frac{1}{3}\pi^3$ to 10 decimal places by hand ... ■

4. Compute $\int_{-\infty}^{\infty} e^{-x^2}\,dx$ using `Maple`. *[2]*

SOLUTION.

```
> int(exp(-x^2),x=-infinity..infinity);
```

$$\sqrt{\pi}$$

e^{-x^2} is a function for which there is no nice formula for the antiderivative ... ■

5. Use `Maple` to find $\int_0^{\pi} e^{-x^2}\,dx$ to 10 decimal places. *[1]*

SOLUTION.

```
> evalf(int(exp(-x^2),x=0..Pi));
```

$$0.8862190595$$

I'd *really* rather not try this one by hand by hand ... ■

6. Find the value of t such that $\int_{-t}^{t} e^{-x^2}\,dx = \frac{1}{2}\int_{-\infty}^{\infty} e^{-x^2}\,dx$, also to 10 decimal places, using `Maple`. *[2] [2]*

SOLUTION.

```
> fsolve(int(exp(-x^2),x=-t..t)=(1/2)*(int(exp(-x^2),
                                    x=-infinity..infinity)),t);
```

$$0.4769362762$$

Whew! ■

Note: In the "Classic" mode, `Maple` use `Pi` and `infinity` to name π and ∞, respectively.