Mathematics 1101Y – Calculus I: functions and calculus of one variable
TRENT UNIVERSITY, 2011–2012
Final Examination

Time: *09:00–12:00, on Friday, 14 April, 2011.* *Brought to you by* Стефан Біланюк.

Instructions: Do parts **U**, **V**, and **W**, and, if you wish, part **B**. Show all your work and justify all your answers. *If in doubt about something,* **ask!**

Aids: Calculator; one aid sheet (all sides!); one (1) brain (with $\leq 10^{10^{10}}$ neurons).

Part U. Do all four (4) of **1**–**4**.

1. Compute $\frac{dy}{dx}$ as best you can in any *three* (3) of **a**–**f**. *[15 = 3 × 5 each]*

a. $\ln(x+y) = 1$ **b.** $y = x\arctan(x)$ **c.** $y = \sin\left(x^2\right)$

d. $\begin{matrix} y = t^2 - 1 \\ x = 2t + 1 \end{matrix}$ **e.** $y = \int_0^x w^\pi \, dw$ **f.** $y = \frac{x^2-1}{x^2+1}$

2. Evaluate any *three* (3) of the integrals **a**–**f**. *[15 = 3 × 5 each]*

a. $\int \frac{1}{\sqrt{9-x^2}}\,dx$ **b.** $\int_0^\infty e^{-x}\,dx$ **c.** $\int \frac{e^s}{e^{2s}+1}\,ds$

d. $\int_1^e \ln(x)\,dx$ **e.** $\int \frac{3x-3}{x^2+x-2}\,dx$ **f.** $\int_0^{\pi/4} \tan(t)\,dt$

3. Do any *three* (3) of **a**–**f**. *[15 = 3 × 5 each]*

a. Evaluate $\lim_{x\to 0} \frac{x}{\cos(x)}$ or show that the limit does not exist.

b. Find the area of the region enclosed by the polar curve $r = \sin(\theta)$ for $0 \leq \theta \leq \pi$. Sketch it, too!

c. Determine whether the series $\sum_{n=0}^{\infty} \frac{1}{2^n+n}$ converges or diverges.

d. Use the Right-hand Rule to compute the definite integral $\int_2^3 x\,dx$.

e. Find the arc-length of the curve $y = \frac{2}{3}x^{3/2}$, $0 \leq x \leq 1$.

f. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{n}$.

4. Consider the region between $y = \sqrt{x}$ and $y = x$ for $0 \leq x \leq 1$.

a. Sketch this region. *[1]*

b. Sketch the solid obtained by revolving this region about the y-axis. *[1]*

c. Find the volume of this solid. *[8]*

Part V. Do any *two* (2) of **5–7**. *[30 = 2 × 15 each]*

5. The area of a rectangle with sides of lengths a and b is ab. Suppose the area of the rectangle is growing at a rate of 10 cm^2/s and a is increasing at a rate of 1 cm/s.

- **a.** How is b changing at the instant that $a = 10$ cm and $b = 20$ cm? *[10]*
- **b.** Suppose b is increasing at a rate of 1 cm/s at some instant. Find possible values for a and b at this instant or show that there are no such values. *[5]*

6. Find all the intercepts, maximum, minimum, and inflection points, and all the vertical and horizontal asymptotes of $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$, and sketch its graph.

7. Sketch the solid obtained by revolving the region between $y = \cos(x)$ and $y = \sin(x)$, for $\dfrac{\pi}{4} \leq x \leq \dfrac{5\pi}{4}$, about the y-axis and find its volume using cylindrical shells.

Part W. Do *one* (1) of **8** or **9**. *[15 = 1 × 15 each]*

8. Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+2}x^n}{n!}$.

- **a.** Find the radius and interval of convergence of this power series. *[10]*
- **b.** What function has this power series as its Taylor series at 0? *[5]*

9. Let $f(x) = \dfrac{1}{2x - 1}$.

- **a.** Use Taylor's formula to find the Taylor series at 0 of $f(x)$. *[8]*
- **b.** Find the Taylor series at 0 of $f(x)$ without using Taylor's formula. *[2]*
- **c.** Find the radius and interval of convergence of this Taylor series. *[5]*

[Total = 100]

Part B. ~~Bear~~ Bonus problems! Do them (or not), if you feel like it.

2b. Suppose a number of circles are drawn on a piece of paper, dividing it up into regions whose borders are made up of circular arcs. Show that one can always colour these regions using only black and white so that no two regions that have a border arc in common have the same colour. *[2]*

¬2b. Write a haiku touching on calculus or mathematics in general. *[2]*

haiku?

seventeen in three:
five and seven and five of
syllables in lines

I HOPE THAT YOU ENJOYED THE COURSE. (REALLY! :-)
HAVE A FUN SUMMER!