Mathematics 1101Y – Calculus I: Functions and calculus of one variable
TRENT UNIVERSITY, 2011–2012

Assignment #2
Off on a poetical tanchent ...
Due on Thursday, 10 November, 2011.

Mathematicians in medieval India traditionally wrote up much of their work in verse. For example, here is a problem* posed by Bhaskara† (*c.* 1114-1185 A.D.) in a book dedicated to his daughter Lilavati:

> The square root of half the number of bees in a swarm
> Has flown out upon a jasmine bush;
> Eight ninths of the swarm has remained behind;
> And a female bee flies about a male who is buzzing inside a lotus flower;
> In the night, allured by the flower's sweet odour, he went inside it
> And now he is trapped!
> Tell me, most enchanting lady, the number of bees.

For those interested in the history of mathematics, Bhaskara developed a number of techniques that anticipated portions of both differential and integral calculus.

1. Restate the problem given above as an equation. *[1]*

2. Solve the equation you obtained in **1** by hand. *[1]*

3. Solve the equation you obtained in **1** using `Maple`. *[1]*

Note: The basic tool you will need to do **3** is `Maple`'s `solve` command, which has many options and variations. Make sure to ask for help if you need it!

Recall that the basic *hyperbolic trigonometric functions*‡ include:

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

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* This translation of Bhaskara's problem is given in *The Heritage of Thales*, by W.S. Anglin & J. Lambeck, Springer Verlag, New York, 1995, ISBN 0-387-94544-X, p. 113.

† There was an earlier notable Indian mathematician also named Bhaskara (*c.* 600-680 A.D.). They sometimes end up being numbered to distinguish them ...

‡ They are so named because they relate angles to side length for triangles in the hyperbolic plane, just as the ordinary trigonometric functions do for triangles in the Euclidean plane.The hyperbolic plane is like the Euclidean plane except that parallel lines work differently: instead of having just one line parallel to a given line through any point not on the given line, there are infinitely many lines parallel to the given line through any point not on the given line. (One annoying consequence is that all those lines through a point parallel to a given line are not parallel to each other.) Hyperbolic geometry, and other non-Euclidean geometries, actually have uses. For one example, the key idea in general relativity is that mass and energy affect the curvature of space, giving it a non-Euclidean geometry.

(Do I like footnotes way too much? Why, yes! Yes, I do! :-)

There are connections between the hyperbolic and the regular trigonometric functions, some of which will become apparent when we study series. Your main task in this assignment will be to investigate $\tanh(x)$ and find its inverse.

4. Use `Maple` to graph $\tanh(x)$. *[1]*

5. From your graph from **4**, what restrictions does the output of $\tanh(x)$ satisfy? Using the definition of $\tanh(x)$ given above, explain why its output has to satisfy these restrictions. *[1]*

6. Use `Maple`'s ability to solve equations symbolically to find an expression for $\operatorname{arctanh}(x)$, the inverse function of $\tanh(x)$.** *[3]*

7. Work out an expression for $\operatorname{arctanh}(x)$ yourself. (If this is different from what `Maple` gave you in **6**, you may well be correct, but try to explain, if you can, why they amount to the same thing.) *[2]*

Bonus. What does Bhaskara's problem have to do with a *Monty Python* sketch? *[0.5]*

Equation Limericks

A *limerick* is a poem with five lines. The first, second, and fifth lines should have nine syllables each and rhyme with each other, and the third and fourth lines should have six syllables each and rhyme with each other. Limericks are usually intended to be funny, and spelling, pronounciation, and grammar are often mangled a little when composing limericks in English. A well-known example of a limerick, relating to physics, is the following:

There was a young lady named Bright,
Who traveled much faster than light.
She started one day
In the relative way,
And returned on the previous night.

By Hellen Barton Tuttle, or A.H. Reginald Buller, F.R.S., or Anonymous ...

An obscure subtype of the limerick is the equation limerick, which states an equation. Here is an example:

$$\left(12 + 144 + 20 + 3 \cdot \sqrt{4}\right)/7 + 5 \cdot 11 = 9^2$$

a dozen, a gross, plus a score
plus three times the square root of four
divided by seven
plus five times eleven
is nine squared (and not a bit more)

Posted to `sci.math` *by Rajeev Krishnamoorthy in 1992.*

Assignment π, an extra assignment which you will get to do over the Christmas break, will involve writing an original equation limerick. You can start to work on it now!

** Our textbook uses the notation $\tanh^{-1}(x)$ for the inverse of $\tanh(x)$.