Maximums and Minimums

Recall that if a differentiable function has a maximum or minimum at a, then

Caveat: It’s to have a critical point without it being a max or a min.

Example 1:

Find all the maxima & minima of on

Differentiate:

Critical Points:

When

When or

How do we check whether these are maxima, or minima?

Build a table...

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x |  | 0 | (0,2) | 2 |  |
|  | - | 0 | + | 0 | - |
|  | Decreasing | Local  Minimum | Increasing | Local  Maximum | Decreasing |

Are these absolute maxima & minima?

Which wins or do they cancel each other out?

The max/min’s are strictly local because .................................

Example 2: Find the maxima & minima of

On

Differentiate:

Critical Points:

When

(Note: is never 0 so the derivative is always defined everywhere)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x |  | (-4, 1) | 1 | (1, 8) |  |
|  |  | + | 0 | - |  |
|  | Min | Increasing | ~~Local~~  Absolute  Maximum | Decreasing |  |

Example 3: Find the Maxima & minima of

On

(Note: that ln(x) are undefined for)

So we’ll do it on the interval instead

Differentiate:

|  |  |
| --- | --- |
| Critical Points:    As is always positive this happens When:        There is no such x. If there are no critical points we look at the end points. | Do & cross  Do &  Intersect for some?  ?  Facts: As you go to grows faster than any polynomial.  As you go to goes to 0 faster than any |

|  |  |  |
| --- | --- | --- |
| x | (0, e) | e |
|  | + |  |
|  | inc | Max (on the given interval) |

Example 4:

On

Differentiate:

Critical Points

when or

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x |  |  | ( |  |  | 0 | (0, 1) | 1 |  |  |  |
|  | - | 0 | + | undef | + | 0 | + | undef | + | 0 | - |
|  | Dec | Min | Inc | asymptote | Inc | Nothing | Inc | asymptote | Inc | Max | Dec |

Mean Value Theorem & Consequence

Graph of segment of a function.

Average slope of y=f(x) on [a, b] is

Is the actual slope ever equal to the average slope.

Yes at some points.

MVT

Suppose f(x) is continuous on some interval [a, b] and differentiable on (a, b). Then there is at least one point c with c strictly between a and b such that

Corollary 1:

Suppose f(x) is a function such that for all c in (a, b).

F(x) is constant (a horizontal graph in this case)

Corollary 2: Suppose g(x) and f(x) are differentiable on (a, b) and for all points c in this interval,

Then on (a, b)

For some constant k

Why?

Then

By the first corollary (k a constant)

Sorting out asymptotes! (vertical and horizontal)

Extra tool for computing limits: 1) Hopital’s Rule

Suppose f(x), g(x) are functions such that &

Then provided this exists

Example 1:

Since &

As x

We can use l’Hopital’s Rule

=

Example 2:

l’Hopital’s Rule

Example 3:

Does have a horizontal asymptote?

Check

:

:

Graphing with calculus

Graph on the whole real line

1. Domain of Wherever is sensible, namely when

Ie: when

1. Intercepts of

y-intercept: set

x-intercept: set y = 0

=> =>

1. Max/Min, critical pts, etc.

Critical points

When and when is undefined

when & is undefined when

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x |  | -1 | (-1, 0) | 0 | (0, 1) | 1 |  |
|  | - | undefined | - | 0 | + | undefined | + |
|  | decreasing | asymptote | decreasing | local min | increasing | asymptote | increasing |

1. Vertical asymptotes

* Possibly at
* Use limits at these points to check.

* top approaches 1
* bottom first part + second part –

* top approaches 1
* bottom first part and second part +

1. Horizontal asymptotes

1. Draw the graph

* Put in the asymptotes
* Put in the critical points