Mathematics 1101Y – Calculus I: functions and calculus of one variable
TRENT UNIVERSITY, 2010–2011

Assignment #11
Parameters in polar coordinates, oh my!
Due on Friday, 25 March, 2011.

We will investigate a curve in polar coordinates for which both r and θ are given by functions of a parameter t:

$$r = \sqrt{5 + 4\cos(t)} \text{ and } \theta = \arctan\left(\frac{\sin(t)}{2 + \cos(t)}\right), \text{ where } 0 \leq t \leq 2\pi.$$

For example, if $t = \frac{2\pi}{3}$, then $\cos(t) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$ and $\sin(t) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$, so $r = \sqrt{5 + 4\cos(t)} = \sqrt{5 + 4\left(-\frac{1}{2}\right)} = \sqrt{5 - 2} = \sqrt{3}$ and $\theta = \arctan\left(\frac{\sin(t)}{2 + \cos(t)}\right) = \arctan\left(\frac{\frac{\sqrt{3}}{2}}{2 + \left(-\frac{1}{2}\right)}\right) = \arctan\left(\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}\right) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$. This point has Cartesian coordinates $x = r\cos(\theta) = \sqrt{3}\cos\left(\frac{\pi}{6}\right) = \sqrt{3}\frac{\sqrt{3}}{2} = \frac{3}{2}$ and $y = r\sin(\theta) = \sqrt{3}\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.

1. Use `Maple` to plot this curve. *[3]*
2. Based on your plot, what is this curve? *[1]*
3. Starting from the given parametric description, work out an equation for this curve in Cartesian coordinates. *[2]*
4. Starting from the given parametric description, work out an equation for this curve in polar coordinates. *[2]*
5. Find the area of the region enclosed by this curve. *[2]*

You may, of course, also use `Maple` to help you with questions **3**–**5**.