Math 1100 — Calculus, Quiz #5B — 2009-11-05

Differentiate the following functions:

(30) 1. $h(x) = \sqrt[5]{x} \cdot \cos(x)$.

Solution: $h(x) = f(x) \cdot g(x)$ where $f(x) = \sqrt[5]{x}$ and $g(x) = \cos(x)$. We have $f'(x) = \dfrac{1}{5x^{4/5}}$ and $g'(x) = -\sin(x)$. Thus, the Leibniz product rule says:

$$h'(x) = f'(x)g(x) + f(x)g'(x) = \boxed{\frac{\cos(x)}{5x^{4/5}} - \sqrt[5]{x}\sin(x).}$$

□

(35) 2. $h(x) = \dfrac{1-\cos(x)}{x+\sin(x)}$.

Solution: $h(x) = \dfrac{f(x)}{g(x)}$, where $f(x) = 1-\cos(x)$ and $g(x) = x+\sin(x)$. We have $f'(x) = \sin(x)$ and $g'(x) = 1+\cos(x)$. Thus, the Quotient Rule says

$$\begin{aligned} h'(x) &= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} = \frac{\sin(x)\cdot(x+\sin(x)) - (1-\cos(x))\cdot(1+\cos(x))}{(x+\sin(x))^2} \\ &= \frac{x\sin(x) + \sin(x)^2 - 1 + \cos(x)^2}{(x+\sin(x))^2} \\ &\underset{(*)}{=} \frac{x\sin(x)+1-1}{(x+\sin(x))^2} = \boxed{\frac{x\sin(x)}{(x+\sin(x))^2}.} \end{aligned}$$

here, $(*)$ is because $\sin(x)^2 + \cos(x)^2 = 1$ by Pythagoras. □

(35) 3. $h(x) = \sqrt[3]{1+\exp(x^3)}$.

Solution: $h(x) = f \circ g(x)$ where $f(y) = \sqrt[3]{y}$ and $g(x) = 1+\exp(x^3)$. We have $f'(y) = \dfrac{1}{3y^{2/3}}$ and $g'(x) = \exp(x^3)\cdot 3x^2$ (by the Chain rule). Thus, a second application of the Chain rule yields

$$h'(x) = f'(g(x))\cdot g'(x) = \boxed{\frac{\exp(x^2)\cdot 3x^2}{3(1+\exp(x^3))^{2/3}}.}$$

□