Math 1100 — Calculus, Quiz #4A — 2009-10-19

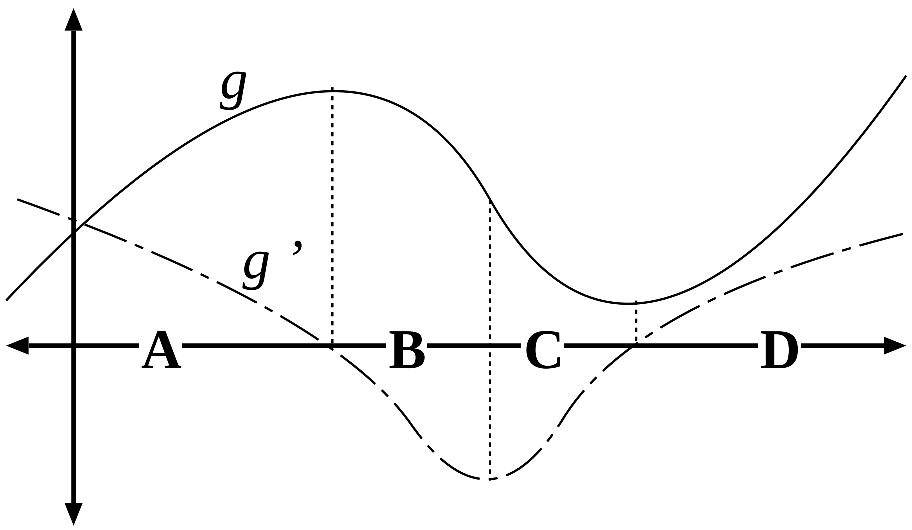
(60) 1. Let $f(x) := 3x^2 - 2x + 1$. Compute the derivative $f'(x)$ using the 'limit' definition of derivative. (Do *not* just apply the 'power rule' to get the answer).

Solution:

$$\begin{aligned} f'(x) &:= \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon} \quad = \quad \lim_{\epsilon \to 0} \frac{(3(x+\epsilon)^2 - 2(x+\epsilon)+1) - (3x^2 - 2x + 1)}{\epsilon} \\ &= \lim_{\epsilon \to 0} \frac{3x^2 + 6x\epsilon + 3\epsilon^2 - 2x - 2\epsilon + 1 - 3x^2 + 2x - 1}{\epsilon} \\ &= \lim_{\epsilon \to 0} \frac{6x\epsilon + 3\epsilon^2 - 2\epsilon}{\epsilon} \\ &= \lim_{\epsilon \to 0} (6x + 3\epsilon - 2) \quad = \quad \boxed{6x - 2.} \end{aligned}$$

□

2. Here is the graph of the function g. Sketch the graph of its derivative g'. In your
(40) sketch, divide the real line into intervals corresponding to regions where g is increasing, decreasing, etc. and relate this to corresponding properties of g'.



Solution: In intervals **A** and **D**, the function g is increasing and g' is positive.

In interval **B** and **C**, the function g is decreasing and g' is negative.

(Bonus) In intervals **A** and **B**, the function g is curving down and g' is decreasing.

In intervals **C** and **D**, the function g is curving up and g' is increasing. □