MATH 1101Y 2009 Quiz 7 (b)

1. Let $f(x) = 2x^3 - 9x^2 - 24x$.

   (a) (2 pts) Find the intervals of increase or decrease.

   (b) (1 pts) Find the local maximum and minimum values.

   (c) (2 pts) Find the intervals of concavity and the inflection points.

Solution: $f'(x) = 6x^2 - 18x - 24$.

Let $f'(x) = 0$. We have

$$\begin{aligned} 6x^2 - 18x - 24 &= 0 \\ x^2 - 3x - 4 &= 0 \\ (x+1)(x-4) &= 0 \end{aligned}$$

$f' = 0$ when $x = -1$ or $x = 4$.

Since

$$\begin{aligned} f'(-2) &= 6(-2)^2 - 18(-2) - 24 \\ &= 36, \end{aligned}$$

$$f'(0) = -24,$$

and

$$\begin{aligned} f'(5) &= 6(5)^2 - 18(5) - 24 \\ &= 36, \end{aligned}$$

$f' > 0$ on $(-\infty, -1) \cup (4, \infty)$ and $f' < 0$ on $(-1, 4)$.

(a) f is increasing on $(-\infty, -1) \cup (4, \infty)$ and decreasing on $(-1, 4)$.

(b) f has a local maximum at $x = -1$ with value $f(-1) = 2(-1)^3 - 9(-1)^2 - 24(-1) = 13$. f has a local minimum at $x = 4$ with value $f(4) = 2(4)^3 - 9(4)^2 - 24(4) = -112$.

$f''(x) = 12x - 18$. Let $f'' = 0$. We have

$$\begin{aligned} 12x - 18 &= 0 \\ x &= \frac{3}{2}. \end{aligned}$$

Since $f''(0) = -18$ and $f''(2) = 6$, $f'' < 0$ on $\left(-\infty, \frac{3}{2}\right)$ and $f'' > 0$ on $\left(\frac{3}{2}, \infty\right)$. f has an inflection point at $x = \frac{3}{2}$ and $y = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 - 24\left(\frac{3}{2}\right) = -\frac{99}{2}$. □