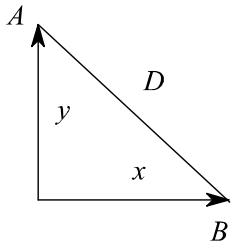
MATH 1101Y 2009 Quiz 6 (a)

1. (3 pts) Two cars start moving from the same point. Car A travels north at 60 km/h and car B travels east at 40 km/h. At what rate is the distance between the cars increasing two hours later?

   Solution:



   Let the distance car B travels be x and the distance car A travels be y. Let the distance between cars A and B be D. We are given that $\frac{dx}{dt} = 40$, $\frac{dy}{dt} = 60$. Since $D = \sqrt{x^2 + y^2}$ and $x = 80$, $y = 120$ two hours later, we have

$$\begin{aligned} \frac{dD}{dt} &= \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2\sqrt{x^2+y^2}} \\ &= \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}} \\ &= \frac{80 \cdot 40 + 120 \cdot 60}{\sqrt{80^2 + 120^2}} \\ &= 20\sqrt{13} \end{aligned}$$

   The distance is changing at $20\sqrt{13} \approx 72.11$ km/h two hours later. □

2. (2 pts) Find the derivative of $f(x) = \sinh(1 + e^{3x})$.

   Solution:

$$\begin{aligned} f'(x) &= \cosh\left(1 + e^{3x}\right) \cdot e^{3x} \cdot 3 \\ &= 3e^{3x} \cosh\left(1 + e^{3x}\right). \end{aligned}$$

□