MATH 1101Y 2009 Quiz 5 (b)

1. (1.5 pts) Find $\frac{dy}{dx}$ by implicit differentiation.

$$\sqrt{x^2+y^2} = \sin\left(xy^2\right)$$

*Solution*: Differentiate both sides. We have

$$\begin{aligned}
\frac{1}{2\sqrt{x^2+y^2}}\left(2x+2y\frac{dy}{dx}\right) &= \cos\left(xy^2\right)\left(y^2+2xy\frac{dy}{dx}\right) \\
\frac{x}{\sqrt{x^2+y^2}} + \frac{y}{\sqrt{x^2+y^2}}\frac{dy}{dx} &= y^2\cos\left(xy^2\right) + 2xy\cos\left(xy^2\right)\frac{dy}{dx} \\
\left(\frac{y}{\sqrt{x^2+y^2}} - 2xy\cos\left(xy^2\right)\right)\frac{dy}{dx} &= y^2\cos\left(xy^2\right) - \frac{x}{\sqrt{x^2+y^2}}
\end{aligned}$$

$$\frac{dy}{dx} = \frac{y^2\cos\left(xy^2\right) - \frac{x}{\sqrt{x^2+y^2}}}{\frac{y}{\sqrt{x^2+y^2}} - 2xy\cos\left(xy^2\right)}.$$

□

2. (1.5 pts) Find $\frac{dy}{dx}$.

$$y = (\ln x)^x$$

*Solution*:

$$\ln y = \ln\left((\ln x)^x\right) = x\ln(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = \ln\ln x + x\frac{1}{\ln x}\frac{1}{x} = \ln\ln x + \frac{1}{\ln x}$$

$$\begin{aligned}
\frac{dy}{dx} &= y\left(\ln\ln x + \frac{1}{\ln x}\right) \\
&= (\ln x)^x\left(\ln\ln x + \frac{1}{\ln x}\right).
\end{aligned}$$

□

3. (2 pts) Find $\frac{dy}{dx}$.

$$y = \sqrt{\frac{x^3+4x^2-x+5}{3x^2+4x-1}}\sqrt[3]{\frac{\tan x}{e^{5x}}}$$

*Solution*:

$$\begin{aligned}
y &= \left(\frac{x^3+4x^2-x+5}{3x^2+4x-1}\right)^{\frac{1}{2}} \cdot \left(\frac{\tan x}{e^{5x}}\right)^{\frac{1}{3}} \\
\ln y &= \frac{1}{2}\left(\ln\left(x^3+4x^2-x+5\right) - \ln\left(3x^2+4x-1\right)\right) \\
&\quad + \frac{1}{3}\left(\ln(\tan x) - \ln\left(e^{5x}\right)\right)
\end{aligned}$$

$$
\begin{aligned}
\frac{1}{y}\frac{dy}{dx} &= \frac{1}{2}\left(\frac{3x^2+8x-1}{x^3+4x^2-x+5} - \frac{6x+4}{3x^2+4x-1}\right) \\
&\quad + \frac{1}{3}\left(\frac{\sec^2 x}{\tan x} - 5\right) \\
\frac{dy}{dx} &= \sqrt{\frac{x^3+4x^2-x+5}{3x^2+4x-1}}\sqrt[3]{\frac{\tan x}{e^{5x}}} \\
&\quad \cdot \left[\frac{1}{2}\left(\frac{3x^2+8x-1}{x^3+4x^2-x+5} - \frac{6x+4}{3x^2+4x-1}\right) + \frac{1}{3}\left(\frac{\sec^2 x}{\tan x} - 5\right)\right].
\end{aligned}
$$

□