MATH 1101Y 2009 Quiz 3 (b)

1. (2 pts) Let

$$f(x) = \begin{cases} 2^x & \text{if } x < 1 \\ \frac{x+2}{x} & \text{if } 1 \le x < 2 \\ \sqrt{x+2} & \text{if } x \ge 2 \end{cases}$$

Find the number at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither?

Solution:

$$\begin{aligned} \lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} 2^x = 2 \\ \lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+} \frac{x+2}{x} = 3 \end{aligned}$$

Therefore $\lim_{x\to 1} f(x)$ does not exist and f is discontinuous at 1. Since $\lim_{x\to 1^+} f(x) = f(1)$, f is continuous from the right at 1.

$$\begin{aligned} \lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} \frac{x+2}{x} = 2 \\ \lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} \sqrt{x+2} = 2 \end{aligned}$$

Therefore $\lim_{x\to 2} = 2 = f(2)$. f is continuous at 2. □

2. (3 pts) Use the definition of derivative to find the derivative of $f(x) = \sqrt{x+3}$.

Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} \\ &= \lim_{h\to 0} \frac{\left(\sqrt{x+h+3} - \sqrt{x+3}\right)\left(\sqrt{x+h+3} + \sqrt{x+3}\right)}{h\left(\sqrt{x+h+3} + \sqrt{x+3}\right)} \\ &= \lim_{h\to 0} \frac{\left(\sqrt{x+h+3}\right)^2 - \left(\sqrt{x+3}\right)^2}{h\left(\sqrt{x+h+3} + \sqrt{x+3}\right)} \\ &= \lim_{h\to 0} \frac{(x+h+3) - (x+3)}{h\left(\sqrt{x+h+3} + \sqrt{x+3}\right)} = \lim_{h\to 0} \frac{h}{h\left(\sqrt{x+h+3} + \sqrt{x+3}\right)} \\ &= \lim_{h\to 0} \frac{1}{\sqrt{x+h+3} + \sqrt{x+3}} = \frac{1}{2\sqrt{x+3}}. \end{aligned}$$

□