MATH 1101Y 2009 Quiz 3 (a)

1. (2 pts) Let

$$f(x) = \begin{cases} 1 + x^2 & \text{if } x < 1 \\ \sqrt{x+3} & \text{if } 1 \le x < 6 \\ \frac{x+1}{2} & \text{if } x \ge 6 \end{cases}$$

Find the number at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither?

Solution:

$$\begin{aligned} \lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} 1 + x^2 = 2 \\ \lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+} \sqrt{x+3} = 2 \end{aligned}$$

Therefore $\lim_{x\to 1} f(x) = 2 = f(2)$. $f(x)$ is continuous at 2.

$$\begin{aligned} \lim_{x\to 6^-} f(x) &= \lim_{x\to 6^-} \sqrt{x+3} = 3 \\ \lim_{x\to 6^+} f(x) &= \lim_{x\to 6^+} \frac{x+1}{2} = \frac{7}{2} \end{aligned}$$

Therefore $\lim_{x\to 6} f(x)$ does not exist. $f(x)$ is discontinuous at 6. Since $\lim_{x\to 6^+} f(x) = f(6)$, f is continuous from the right at 6. □

2. (3 pts) Use the definition of derivative to find the derivative of $f(x) = \frac{x+2}{x+3}$.

Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{x+h+2}{x+h+3} - \frac{x+2}{x+3}}{h} \\ &= \lim_{h\to 0} \frac{\frac{(x+h+2)(x+3)-(x+2)(x+h+3)}{(x+h+3)(x+3)}}{h} \\ &= \lim_{h\to 0} \frac{(3h + 5x + hx + x^2 + 6) - (2h + 5x + hx + x^2 + 6)}{(x+h+3)(x+3)h} \\ &= \lim_{h\to 0} \frac{h}{(x+h+3)(x+3)h} = \lim_{h\to 0} \frac{1}{(x+h+3)(x+3)} \\ &= \frac{1}{(x+3)^2}. \end{aligned}$$

□