MATH 1101Y 2009 Quiz 18 (a)

1. (1 pts) Determine whether the sequence converges or diverges. If it converges, find the limit.
$$a_n = \frac{4^{n+3}}{5^n}$$

   Solution:
$$\begin{aligned} \lim_{n\to\infty} \frac{4^{n+3}}{5^n} &= \lim_{n\to\infty} 4^3 \frac{4^n}{5^n} \\ &= \lim_{n\to\infty} 4^3 \left(\frac{4}{5}\right)^n = 0 \end{aligned}$$

   The sequence converge. The limit is 0. □

2. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

   (a) (1 pts)
$$\sum_{k=1}^{\infty} \frac{k}{2k+1}$$

   Solution: Since
$$\lim_{k\to\infty} \frac{k}{2k+1} = \frac{1}{2} \neq 0$$
   the series is convergent by the Divergence Test. □

   (b) (1 pts)
$$\sum_{n=1}^{\infty} \frac{1}{3n}$$

   Solution: We have
$$\sum_{n=1}^{\infty} \frac{1}{3n} = \frac{1}{3} \sum_{n=1}^{\infty} \frac{1}{n}.$$
   The series is divergent since the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent. □

   (c) (2 pts)
$$\sum_{n=1}^{\infty} \frac{3^n - 1}{5^n}$$

   Solution:
$$\begin{aligned} \sum_{n=1}^{\infty} \frac{3^n - 1}{5^n} &= \sum_{n=1}^{\infty} \frac{3^n}{5^n} - \sum_{n=1}^{\infty} \frac{1}{5^n} \\ &= \sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n - \sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n \end{aligned}$$

The series is convergent since both geometric series have $r < 1$ ($r = \frac{3}{5}$ and $\frac{1}{5}$) respectively. The sum is

$$\begin{aligned}
\frac{\frac{3}{5}}{1-\frac{3}{5}} - \frac{\frac{1}{5}}{1-\frac{1}{5}} &= \frac{\frac{3}{5}}{\frac{2}{5}} - \frac{\frac{1}{5}}{\frac{4}{5}} \\
&= \frac{3}{2} - \frac{1}{4} = \frac{5}{4}.
\end{aligned}$$

□