MATH 1101Y 2009 Quiz 17 (b)

1. (1 pts) Identify the curve by finding a Cartesian equation for the curve

$$r = 6\cos\theta - \sin\theta$$

Solution:

$$\begin{aligned} r^2 &= 6r\cos\theta - r\sin\theta \\ x^2 + y^2 &= 6x - y \\ x^2 - 6x + y^2 + y &= 0 \\ x^2 - 6x + 9 + y^2 + y + \left(\frac{1}{2}\right)^2 &= 9 + \frac{1}{4} = \frac{37}{4} \\ (x-3)^2 + \left(y + \frac{1}{2}\right)^2 &= \frac{37}{4} \end{aligned}$$

The curve is a circle with center $\left(3, -\frac{1}{2}\right)$ and radius $\sqrt{\frac{37}{4}}$. □

2. (2 pts) Find the (x, y)-coordinates of the points on the curve $r = 5\sin\theta$ where the tangent line is horizontal or vertical.

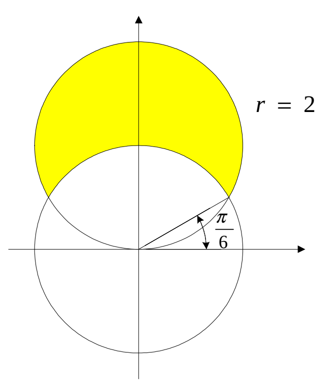
Solution:

$$\begin{aligned} \frac{dy}{d\theta} &= 10\sin\theta\cos\theta \\ \frac{dx}{d\theta} &= 5(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) \end{aligned}$$

$\frac{dy}{d\theta} = 0$ when $\theta = 0, \pi, \pm\frac{\pi}{2}$. $\frac{dx}{d\theta} = 0$ when $\tan\theta = \pm 1$, $\theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$. □

3. (2 pts) Set up an integral that represents the area of the region that is inside $r = 2\sin\theta$ and outside $r = 1$. Do not evaluate this integral.

Solution:



We let

$$\begin{aligned} 2\sin\theta &= 1 \\ \sin\theta &= \frac{1}{2} \end{aligned}$$

Therefore, $\theta = \frac{\pi}{6}$ or $\theta = \frac{5\pi}{6}$ and between $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$, $2\sin\theta > 1$, the area is

$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}\left((2\sin\theta)^2 - 1\right) d\theta$$

□