

MATH 1101Y 2009 Quiz 14 (b)

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

1. $\int_1^\infty (1 + e^{-5x}) dx$.

Solution:

$$\begin{aligned} \int_1^\infty (1 + e^{-5x}) dx &= \lim_{b \rightarrow \infty} \int_1^b (1 + e^{-5x}) dx \\ &= \lim_{b \rightarrow \infty} \left[x - \frac{e^{-5x}}{5} \right]_1^b \\ &= \lim_{b \rightarrow \infty} \left[\left(b - \frac{e^{-5b}}{5} \right) - \left(1 - \frac{e^{-5}}{5} \right) \right] \\ &= \infty \end{aligned}$$

The integral is divergent.

□

2. $\int_2^\infty \frac{1}{x(\ln x)^2} dx$.

Solution:

$$\begin{aligned} \int_2^\infty \frac{1}{x(\ln x)^2} dx &= \lim_{b \rightarrow \infty} \int_2^b \frac{1}{x(\ln x)^2} dx \quad (\text{Let } u = \ln x. \quad du = \frac{1}{x} dx.) \\ &= \lim_{b \rightarrow \infty} \int_{\ln 2}^{\ln b} \frac{1}{u^2} du = \lim_{b \rightarrow \infty} \left[-\frac{1}{u} \right]_{\ln 2}^{\ln b} \\ &= \lim_{b \rightarrow \infty} \left(-\frac{1}{\ln b} + \frac{1}{\ln 2} \right) = \frac{1}{\ln 2}. \end{aligned}$$

The integral is convergent.

□